Grid filter models for large-eddy simulation

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A new interpretation of approximate deconvolution models (ADM) when used with implicit filtering as a way to approximate the projective grid filter is given. Consequently, a new category of subgrid models, the grid filter models (GFM) is defined. ADM appear as a particular case of GFM since only approximate deconvolution is achieved.

1 Introduction

Approximate deconvolution models (ADM), based on the van Cittert method, were introduced by Stolz and Adams [1] as a particular family of subgrid models for large-eddy simulation. These models rely on the attempt to recover, at least partially, the original unfiltered fields by inverting the filtering operator applied to the Navier–Stokes equations. In the case of isothermal flows of Newtonian incompressible fluids, the LES governing equations for the filtered quantities denoted by an *overbar*, obtained by applying a convolution filter $\mathcal{G}\star$ to the Navier–Stokes equations, read formally

$$\frac{\partial \overline{\mathbf{u}}}{\partial t} + \mathbf{f}(\overline{\mathbf{u}}) = [\mathbf{f}, \mathcal{G}\star](\mathbf{u}), \qquad \text{with} \quad [\mathbf{f}, \mathcal{G}\star](\mathbf{u}) \text{ is the subgrid commutator.} \tag{1}$$

2 Grid filter models

The previous development do not take explicitly into account the major role played by the grid filter in practical LES. Indeed, the filter $\mathcal{G}\star$ to consider in practical simulations is a composition of the convolution filter $\mathcal{L}\star$ and the projective grid filter, referred to as $\mathcal{P}\star$ such that $\mathcal{G}\star = (\mathcal{L} \circ \mathcal{P})\star$, thereby embodying the explicit LES filter $\mathcal{L}\star$ and the implicit projective grid filter $\mathcal{P}\star$ [2, 3], represented by a *hat*. In the sequel, the focus is put on the particular case where implicit grid filtering is the only effective filter. The objective is to give a theoretical interpretation allowing to use a deconvolution model in this framework. When no LES filter $\mathcal{L}\star$ is explicitly applied, the filtered Navier–Stokes equations reduces to

$$\frac{\partial \hat{\mathbf{u}}}{\partial t} + \mathbf{f}(\hat{\mathbf{u}}) = [\mathbf{f}, \mathcal{P}\star](\mathbf{u}), \tag{2}$$

where $\hat{\mathbf{u}}$ is the grid filtered velocity, or in other words the part of the velocity field that can be resolved by the grid used to perform the LES. In this framework, subgrid modeling based on deconvolution models requires the subgrid commutator of the previous equation to be expressed solely in terms of the known projected velocity $\hat{\mathbf{u}}$, which formally reads

$$[\mathbf{f}, \mathcal{P}\star](\mathbf{u}) = [\mathbf{f}, \mathcal{P}\star](\mathcal{P}^{-1} \star \hat{\mathbf{u}}), \qquad \text{with} \quad \mathbf{u} = \mathcal{P}^{-1} \star \hat{\mathbf{u}}, \tag{3}$$

is the formal inverse grid filtering operation to be devised. It is important adding here that due to its implicit nature, the grid filter $\mathcal{P}\star$ entirely depends on the nature of the numerical method used to discretize in space the Navier–Stokes equations.

The problem with Eq. (3) is that $\mathcal{P}^{-1}\star$ does not exist, this filter being a projector. The central idea introduced in Bouffanais [4, 5] is then to approximate $\mathcal{P}\star$ with an invertible filter reproducing as closely as possible its effect

$$\mathcal{P}\star \simeq \mathcal{M}\star \quad \Rightarrow \quad [\mathbf{f}, \mathcal{P}\star](\mathbf{u}) \simeq [\mathbf{f}, \mathcal{M}\star](\mathcal{M}^{-1} \star \hat{\mathbf{u}}). \tag{4}$$

For such subgrid models, modeling only requires to build $\mathcal{M}\star$ in order to achieve the best approximation of $\mathcal{P}\star$, as stated in Eq. (4). The problem is exactly solved if $\mathcal{P}\star = \mathcal{M}\star$, which is not achievable since $\mathcal{P}\star$ applies on an infinite spectrum while $\mathcal{M}\star$ only applies to computable wave numbers [6]. Moreover, $\mathcal{M}\star$ must be invertible and $\mathcal{P}\star$ is projective. The approximation (4) being the only required modeling, the subgrid models arising from this new approach are referred to as grid filter models (GFM). The new interpretation of deconvolution models resulting from the definition of GFM builds a theoretical basis that allows the use of such subgrid models without any explicit filtering of the solution. Moreover, this viewpoint allows to consider a new way of designing the filter $\mathcal{M}\star$ which has to approximate the grid filter $\mathcal{P}\star$ as accurately as possible.

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3 Numerical results

The next step is to perform *a posteriori* validations using GFM. Implicit LES (ILES), i.e. without using any subgrid model, is also presented to emphasize the action of the considered models and to make sure that the space discretization is not refined enough to represent all scales of the solution. LES of the lid-driven cubical cavity flow presented here are performed using the Legendre spectral element method (SEM) and refer to the same computational parameters as [7] and the same physical parameters as the DNS performed by Leriche and Gavrilakis [8], taken as the reference solution. The Reynolds number $\mathrm{Re} = 2hU_0/\nu$, where $2h$ is the size of the cavity and U_0 the imposed lid velocity, is equal to 12'000. The filter $\mathcal{M}\star$ used to approximate the grid filter relies on the application of a given low-pass transfer function in a hierarchical modal basis, as reported in [7]. Improvements in the design of the filter $\mathcal{M}\star$ as an approximation of the grid filter will consequently lead to better modeling.

Subgrid modeling in the case of a flow with coexisting laminar, transitional and turbulent zones such as the lid-driven cubical cavity flow for such Reynolds number represents a challenging problem. Kinetic energy is constantly provided to the fluid by the viscous diffusion induced by the lid motion and the confined nature of the flow avoids the evacuation of energy through an outflow section. The coupling of the lid-driven cubical cavity flow problem with the SEM provides a well suited framework to analyze and compare the accuracy of the different subgrid models.

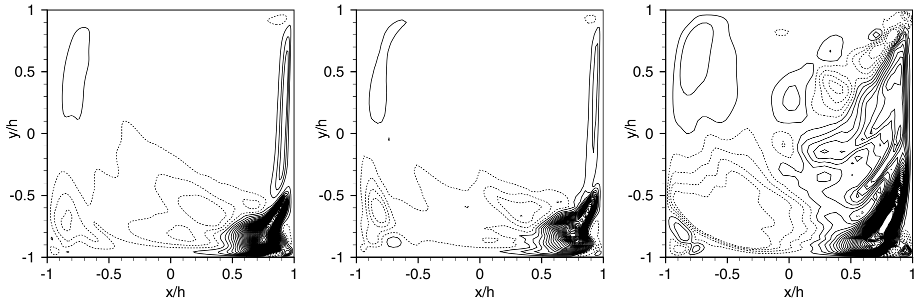

Fig. 1 Contours of $\langle \hat{u}_x^\circ \hat{u}_y^\circ \rangle$ from -0.0007 to 0.0065 in U_0^2 units, by increments of 0.0002 in the midplane $z/h = 0$. DNS (left), GFM (center), ILES (right). Dashed contours correspond to negative levels. The cavity lid $y/h = 1$, moves at U_0 in the x-direction.

To provide an assessment of the performances of the subgrid scales models, the determination of the Reynolds stress tensor components has been envisaged as a challenge in the framework of the lid-driven cubical cavity flow. The Reynolds statistical decomposition $\mathbf{u} = \langle \mathbf{u} \rangle + \mathbf{u}^\circ$, introduces the average value denoted into brackets and its fluctuating part $\mathbf{u}^\circ$. The statistics for all LES are based on a sampling approximately 10 times smaller than the one of the DNS; more precisely 400 samples are collected over $80\, h/U_0$ time units. Figure 1 displays the low-amplitude cross term $\langle \hat{u}_x^\circ \hat{u}_y^\circ \rangle$ in the symmetry plane of the cavity. The contours for ILES show that implicit modeling is totally inoperative in the SEM framework and highlight the need for explicit subgrid modeling. On the other hand, the contours for GFM are in good agreement with those from the DNS.

4 Conclusions

The GFM approach gives a theoretical justification to the use of ADM without explicit filtering of the solution and explains how the use of ADM works in this context. This viewpoint allows to consider a new way of designing the convolution filter which has to approximate the grid filter and therefore a new way of improving such subgrid models. LES of compressible and visco-elastic fluid flows can also be envisaged using GFM. From a numerical viewpoint, GFM can be implemented with all numerical methods allowing filtering operations only needed to compute the subgrid commutator.

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