

The Complexity of Urban Verticality

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Abstract

As cities increasingly develop in three dimensions, their vertical structure becomes an important but underexamined component of urban form. This study introduces a spatial scaling framework for characterizing urban vertical complexity from Digital Surface Model and Digital Terrain Model data. Using a hierarchical quadtree representation, the framework generates a Total Absolute Difference (TAD) profile across successive scale levels and derives the tail slope as the principal comparative metric. The framework is evaluated through nested windows in Canary Wharf, 192 1 km $\times$ 1 km windows across three London zones, and 17 standardized 8 km $\times$ 8 km urban-zone windows in England with their constituent subwindows. Across these analyses, TAD profiles generally show a shallow head followed by a steeper tail, with most variation occurring in the tail regime. Although the tail slope is associated with upper height percentiles, it is not reducible to height alone. The proposed framework therefore provides a quantitative approach for examining urban vertical complexity, enabling comparison of how vertical structure is organized and revealed across different urban landscapes.

Keywords: Urban vertical complexity, urban morphology, spatial scaling, skyline, quadtree

Introduction

High-rise buildings are a defining feature of modern cities, historically symbolizing power and today reflecting economic intensification [1]. With more than half of the world’s population living in urban areas in 2020 [2] and continued global population growth expected, vertical city expansion is often proposed as a strategy for compact [3, 4], transit-oriented [5, 6], and sustainable development [7, 8]. However, the desirability of verticality is a subject of active debate; while it may mitigate horizontal sprawl, it also introduces challenges regarding urban heat islands, pollution exposure, and equitable amenity access. Regardless of these planning trade-offs, the structural complexity of the vertical dimension remains significantly less understood than its 2D horizontal counterpart [9].

To quantitatively capture the verticality of a city, a systematic measurement and analysis of building heights is required. In recent decades, Digital Elevation Model (DEM) data, including Digital Terrain Model (DTM) and Digital Surface Model (DSM), have been frequently used to capture the physical structure of the Earth’s surface [10]. While a DTM represents the bare ground elevation, a DSM is a raster dataset that captures the total height of the urban surface, including buildings and vegetation. At the city scale, DSM datasets can be considered as two-dimensional city skylines or city surfaces, with one dimension representing the East-West direction and the other dimension representing the North-South direction. Since DEM datasets can capture the physical structure of the city surface, they offer new possibilities for analyzing the concept of urban verticality. By leveraging these datasets, researchers and urban planners can gain valuable insights into city surface complex patterns and dimensions, ultimately facilitating more informed decision-making and sustainable urban development.

The physical structure of a city surface displays a complex pattern, with skyscrapers as tall as clouds, streets as low as the ground, and rivers below the ground [11]. Highly urbanized downtown areas often have a greater likelihood of exhibiting complex vertical structure because they combine buildings of different heights, dense development, infrastructure, and open spaces within short distances. This pattern is visible in locations such as London and Manchester (Fig. 1). However, vertical complexity is not determined by building height alone. A uniformly developed high-rise district may exhibit high verticality but comparatively limited structural heterogeneity, whereas an area combining high-, medium-, and low-rise elements may exhibit a more complex vertical pattern. By contrast, predominantly low-rise and relatively homogeneous urban environments, such as York and Southampton, generally display flatter and simpler vertical structures.

Measurements of urban form are sensitive to the spatial units and resolutions through which the landscape is represented. This general issue is related to the Modifiable Areal Unit Problem (MAUP), which describes how analytical results may vary with changes in spatial aggregation or zoning [12–14]. In raster data, progressively aggregating cells changes the amount of spatial detail retained in the representation. Rather than attempting to resolve the MAUP, the present study examines this aggregation process explicitly by measuring how the discrepancy between an aggregated surface and the original high-resolution surface changes across successive scale levels.

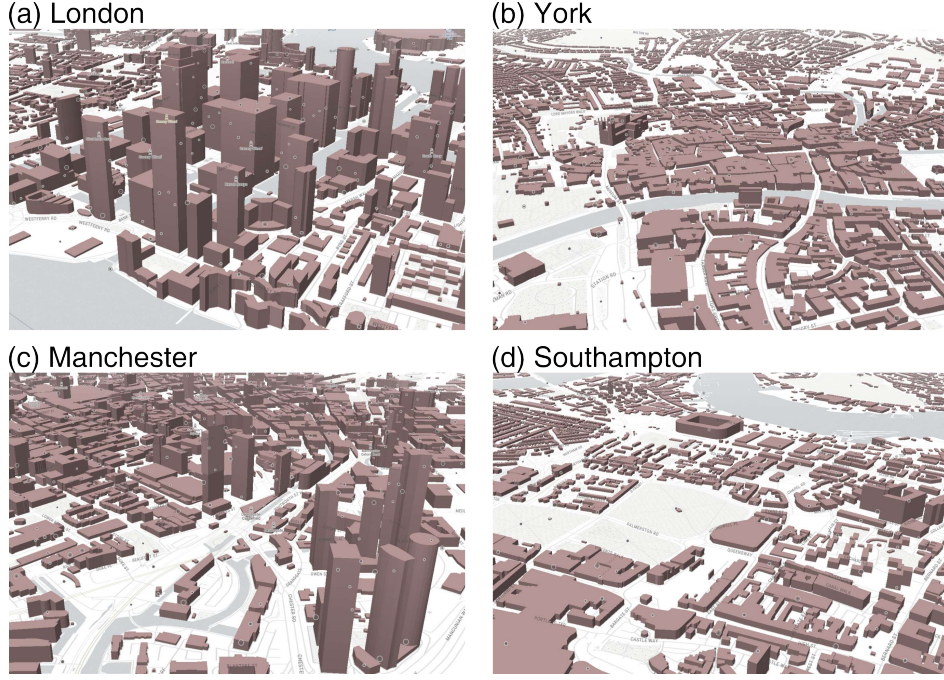


Fig. 1 Comparison of contrasting urban vertical landscapes: a heterogeneous high-rise environment with varied buildings and infrastructure (left) and a more uniform, predominantly low-rise environment (right). Visualizations were created by the authors using Mapbox 3D and OpenStreetMap data with consistent pitch and zoom settings (© Mapbox; © OpenStreetMap contributors). The comparison illustrates that vertical complexity depends on the heterogeneity and spatial arrangement of urban elements rather than on building height alone.

This scale-dependent change provides the basis for characterizing the vertical structure of urban landscapes.

The scaling of urban verticality can be conceptually framed through fractal geometry—a field that studies complex shapes with non-integer dimensions. Unlike Euclidean geometry, where dimensions are integers (1D, 2D, 3D), fractal patterns exhibit a fractional dimension that reflects how detail fills space as the scale of observation changes. In the context of geography, Goodchild and Mark [15] identified three critical axes of research opened by this concept: (1) the response of measure to scale, (2) self-similarity across scales, and (3) patterns observed through the recursive subdivision of space [16–18]. Unlike artificial fractals, which exhibit exact self-similarity, urban forms may exhibit statistical self-similarity, whereby selected structural properties display regularities across ranges of observation scales. This means that while the city does not look identical at every zoom level, selected statistical properties may exhibit regular scaling relationships across a range of scales [19, 20].

Fractal patterns have been observed across a wide array of natural features [17, 21–32] and human-built systems, ranging from street networks and mobility systems to urban boundaries [33–45]. These studies suggest that scaling properties are fundamental to the organization of geographic phenomena.

Urban morphology studies the primary elements of urban form—such as the shape of urban boundaries and building blocks—and the transformation processes that shape human habitats [46, 47]. Fractal analysis has become a fundamental tool in this field [37, 48], applied to topics including population density [49], urban land use [50, 51] and the compactness of urban growth [38, 52]. While the literature suggests that urban verticality should also exhibit these scaling properties, to our knowledge, no systematic fractal analysis of the urban vertical patterns and skylines has been conducted using high-resolution raster data.

Cities evolve as fractal systems where local interactions generate global scaling properties [43, 53]. Fractal approaches have therefore been used to characterize urban form across observation extents, from individual neighborhoods to metropolitan regions [54, 55]. Extending this perspective to the vertical dimension may provide a complementary description of urban morphology that is not captured by conventional two-dimensional measures.

While 2D morphological metrics—such as street connectivity, land-use mix, and building footprints—have been foundational in characterizing urban sprawl and human interaction [39, 56], recent scholarship suggests they remain volumetrically blind, potentially under-representing the intensity of urban interactions and environmental impacts that occur in the vertical dimension [57]. Consequently, there is an increasing recognition that cities appearing identical in 2D planimetric views can possess radically different structural properties and volumetric textures. This highlights a critical need for new analytical frameworks that can bridge the gap between traditional 2D urban morphology and the increasingly complex 3D nature of modern urban systems [9, 58].

In urban form analysis, characterizing structural patterns and understanding the generality of complexity are receiving increasing attention, supported by the growing availability of high-resolution spatial data. Although fractal and scaling properties have been widely examined for two-dimensional urban phenomena, the vertical structure of cities requires similar scrutiny. In this study, we define urban vertical complexity as the rate at which information about the vertical landscape is revealed across successive spatial aggregation levels. Despite its potential importance, the vertical dimension of urban form has rarely been examined from this scaling perspective [9, 58], partly because of the historical lack of high-resolution, city-scale elevation data.

Inspired by these studies, we propose a framework for exploring and quantifying urban vertical complexity using Digital Elevation Model data. Here, an analysis window denotes the geographic extent for which one TAD profile is calculated, whereas scale levels denote the successive quadtree aggregation levels within that window. For each analysis window, the framework calculates the Total Absolute Difference (TAD) between the original high-resolution surface and its aggregated representations across successive scale levels, producing a TAD profile. From the TAD profile, we derive the tail slope as the principal metric of vertical complexity. The tail slope characterizes the rate at which additional vertical information is revealed across increasingly fine aggregation levels, thereby providing a comparative measure of the intensity and heterogeneity of urban vertical structure. This approach offers a quantitative basis for comparing how different urban forms—from high-rise cores to predominantly low-rise areas—manifest across scale levels.

To evaluate the proposed framework across different observation extents, we employ three complementary analyses. First, a nested-window experiment is conducted in Canary Wharf, London, using analysis windows ranging from $125\text{ m} \times 125\text{ m}$ to $1\text{ km} \times 1\text{ km}$. This analysis examines whether the vertical-complexity pattern remains observable across nested analysis windows of different sizes within the same study area. Second, the framework is applied to 192 $1\text{ km} \times 1\text{ km}$ analysis windows across three zones of London to examine the intra-urban distribution of vertical complexity. Third, standardized $8\text{ km} \times 8\text{ km}$ analysis windows and their corresponding $1\text{ km} \times 1\text{ km}$ subwindows are used across 17 urban zones in England to compare both overall vertical-complexity characteristics and within-zone variation among different urban environments. Together, these analyses assess the consistency and interpretability of the proposed metric across nested-window, intra-urban, and cross-zone applications.

Results

Before presenting the results, we distinguish between two concepts used throughout this study: analysis windows and scale levels. An analysis window refers to the spatial extent over which a TAD profile is computed, whereas scale levels refer to the successive quadtree aggregation levels within that window. For each analysis window, a TAD profile is generated across multiple scale levels, from which the head section, tail section, turning point, and corresponding slopes can be identified. The turning point marks the transition from a coarse regime, in which aggregation captures broad variation in the window, to a finer regime in which additional subdivision increasingly resolves localized vertical heterogeneity. The turning point is estimated separately for each analysis because the profile sets and window dimensions differ. Among these characteristics, the tail slope is the primary metric used in this study to quantify the intensity of urban vertical complexity.

The proposed framework is demonstrated using analysis windows of different sizes (Table 1). Three complementary analyses are presented: (1) a nested-window experiment in Canary Wharf; (2) a set of non-overlapping $1\text{ km} \times 1\text{ km}$ windows across three zones of London; and (3) standardized $8\text{ km} \times 8\text{ km}$ windows across 17 urban zones in England. Together, these analyses examine the behavior and interpretation of the proposed vertical-complexity metric across different urban environments and analysis-window sizes.

Table 1 Analysis windows used in the three analyses.

No.	Window designation	Window dimensions	Analyses
1	Urban-zone window	$8\text{ km} \times 8\text{ km}$	3
2	Base 0 window	$1\text{ km} \times 1\text{ km}$	1–3
3	Base 1 window	$500\text{ m} \times 500\text{ m}$	1
4	Base 2 window	$250\text{ m} \times 250\text{ m}$	1
5	Base 3 window	$125\text{ m} \times 125\text{ m}$	1

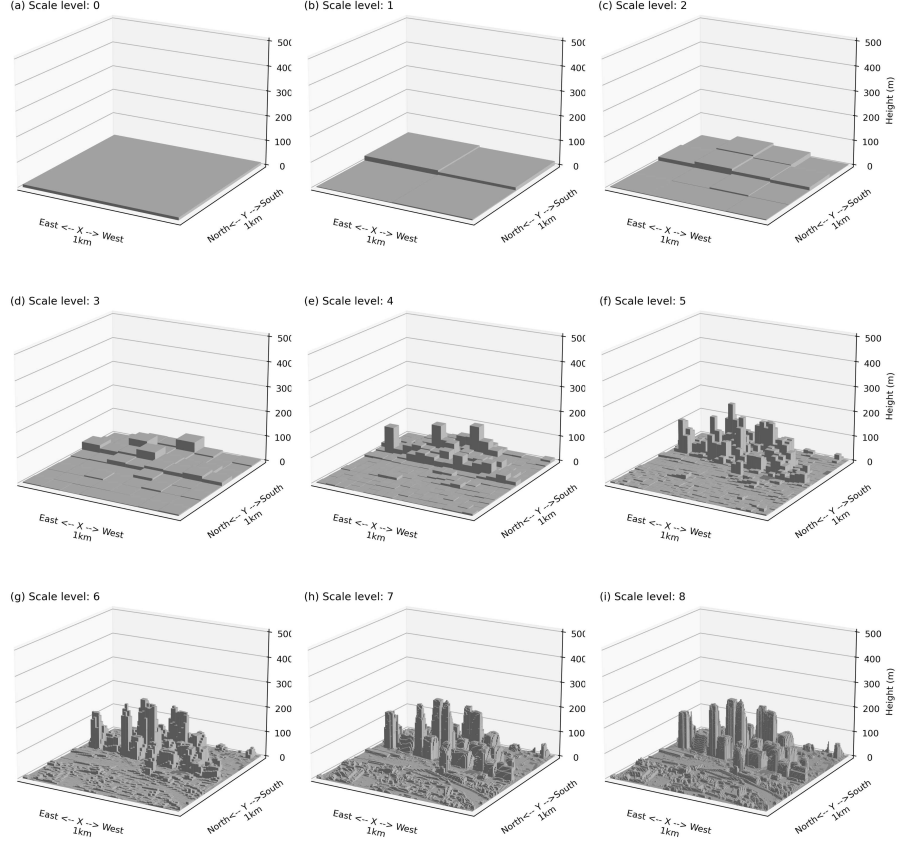


Fig. 2 Illustration of the recursive spatial aggregation framework applied to the 1 km $\times$ 1 km Canary Wharf analysis window. At each scale level, the window is subdivided into smaller cells, within which average surface heights are calculated. The vertical axis uses a different scale from the horizontal plane to make height variation visible. The figure shows how progressively finer scale levels reveal additional detail in the vertical surface.

Analysis 1: Nested-window analysis in Canary Wharf

The first analysis examines whether the tail slope derived from the TAD profile can distinguish vertical landscapes within a hierarchy of nested analysis windows. We use a 1 km $\times$ 1 km area in Canary Wharf (Fig. 2), London's financial district. The southern portion contains a concentration of high-rise buildings, whereas the northern portion is characterized predominantly by low-rise development. The full area is designated as Base 0 and is recursively subdivided into smaller, equal-sized nested analysis windows, designated Base 1 to Base 3 (see Table 1). A TAD profile is generated independently for each analysis window, from which the corresponding tail slope is derived. The four base configurations are: (1) Base 0, comprising one 1 km $\times$ 1 km window; (2) Base 1, comprising four 500 m $\times$ 500 m windows; (3) Base 2, comprising 16 250 m $\times$ 250 m windows; and (4) Base 3, comprising 64 125 m $\times$ 125 m windows.

Figure 3 presents the TAD profiles as a function of scale level s for the four base configurations. Across the analysis windows, the profiles generally exhibit a two-regime decreasing pattern: a relatively gradual decline at lower scale levels, referred to as the head section, followed by a steeper decline at finer scale levels, referred to as the tail section. The transition between the head and tail sections was first identified separately for each TAD profile using an automated gradient-matching procedure based on first-order polynomial fits. This procedure determines the scale level at which the local rate of change becomes more consistent with the later, steeper portion of the profile than with the initial, flatter portion. To provide a consistent basis for comparison among windows within Analysis 1, the most frequently identified transition level was then adopted as the common cutoff. Accordingly, scale level $s = 4$ was used to separate the head and tail sections for all base configurations, as indicated by the black dashed vertical line in Fig. 3.

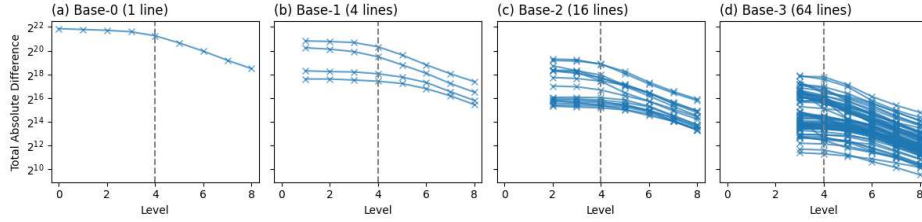


Fig. 3 TAD profiles for the four nested-window configurations in Canary Wharf: (a) Base 0, comprising one profile; (b) Base 1, comprising four profiles; (c) Base 2, comprising 16 profiles; and (d) Base 3, comprising 64 profiles. The black dashed line at scale level $s = 4$ indicates the common cutoff between the head and tail sections. Across the window sizes, the profiles retain a general transition from a shallow head section to a steeper tail section, while finer windows reveal greater local variation.

Because the nested windows at Base 1 to Base 3 begin at progressively finer scale levels, they do not retain all of the coarser levels represented in Base 0. The purpose of this analysis is therefore not to compare head slopes or turning-point positions directly across the four base configurations. Instead, it examines whether the spatial contrast in tail slopes remains observable when the same study area is represented using analysis windows of different sizes. Each tail slope summarizes the scale-dependent vertical structure within its own analysis window and should not be interpreted as one stage in a sequential progression from Base 0 to Base 3.

For Base 0, the full Canary Wharf area is treated as a single analysis window, producing one TAD profile beginning at scale level $s = 0$. This profile summarizes how the discrepancy between the aggregated and original surfaces decreases as progressively finer scale levels are introduced. At Base 1, the area is divided into four equal-sized windows, producing four TAD profiles beginning at $s = 1$. The two profiles corresponding to the southern windows have higher TAD values in the head section and decline more steeply in the tail section than the two profiles corresponding to the northern windows. The southern windows contain the main concentration of high-rise development, whereas the northern windows are characterized predominantly by lower-rise structures (Fig. 4).

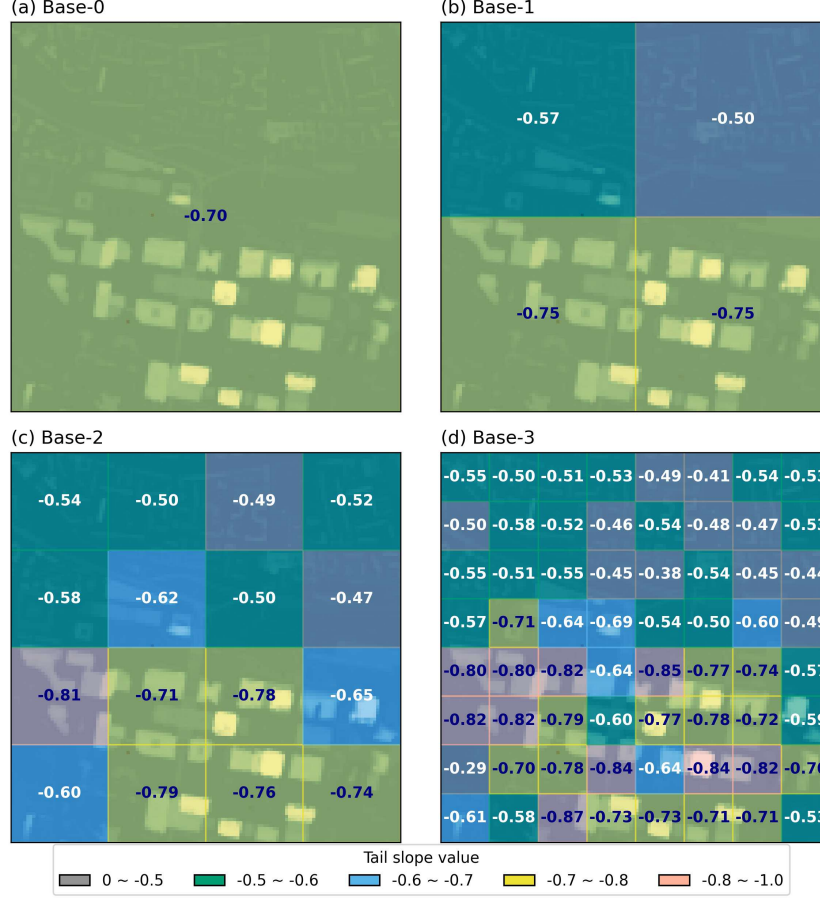


Fig. 4 Normalized Digital Surface Model (nDSM) of Canary Wharf with the fitted tail slope shown for each analysis window: (a) Base 0, (b) Base 1, (c) Base 2, and (d) Base 3. More negative slopes indicate steeper tail sections and greater inferred vertical complexity. Across the nested-window configurations, steeper slopes are concentrated in the vertically heterogeneous southern portion of Canary Wharf, whereas the predominantly low-rise northern portion generally exhibits flatter slopes.

At Base 2, 16 TAD profiles are produced, each beginning at $s = 2$. The profiles associated with the northern windows generally retain the lower initial TAD values and flatter tail behavior observed at Base 1. In contrast, several southern windows exhibit higher TAD values and steeper declines, reflecting stronger variation in the vertical surface across successive scale levels. At Base 3, 64 profiles are produced, beginning at $s = 3$. Although the profiles show greater local variation at this finer window size, the broader contrast between the predominantly low-rise northern area and the more vertically heterogeneous southern area remains visible.

The tail slopes are calculated using linear least-squares regression on the log-transformed TAD profiles and are shown for the corresponding analysis windows in Fig. 4. Across the study area, the fitted tail sections exhibit high coefficients of

determination ($R^2 > 0.89$), indicating that they are closely approximated by linear relationships in log space (Supplementary Fig. S3(a)). For Base 0, the tail slope is -0.70 (Fig. 4(a)). At Base 1 (Fig. 4(b)), the two southern windows have steeper slopes (-0.75) than the two northern windows (-0.50 and -0.57). A similar north-south contrast is observed at Base 2 and Base 3. At Base 2 (Fig. 4(c)), the northern windows have slopes ranging from -0.47 to -0.62 , whereas the southern windows range from -0.61 to -0.81 . At Base 3 (Fig. 4(d)), most northern windows have comparatively flatter slopes (≥ -0.60), while several southern windows associated with concentrations of high-rise buildings exhibit steeper slopes (≤ -0.70).

These results show that the tail slope varies systematically across the nested analysis windows and distinguishes the predominantly low-rise northern portion of Canary Wharf from the more vertically heterogeneous southern portion. The steeper slopes observed in several southern windows indicate that additional vertical information is revealed more rapidly across finer scale levels in these areas. The contrast is associated with the mixture of high-rise buildings, lower structures, streets, and open spaces, rather than with building height alone. Analysis 1 therefore provides an initial demonstration that the tail slope can differentiate vertical landscapes across analysis windows of different sizes.

Analysis 2: Intra-urban analysis across London

The second analysis examines the spatial distribution of vertical complexity within a metropolitan region. The proposed framework is applied independently to 192 non-overlapping $1\text{ km} \times 1\text{ km}$ analysis windows covering the West, Central, and East zones of London (Fig. 5). Each zone comprises 64 windows arranged within a standardized $8\text{ km} \times 8\text{ km}$ extent. A TAD profile and corresponding tail slope are derived for each window, allowing spatial variation in vertical complexity to be compared within and across the three zones. The resulting tail slopes should be interpreted as window-specific summaries for the selected $1\text{ km} \times 1\text{ km}$ analysis windows rather than as estimates of a continuous local surface.

TAD profiles are derived for each analysis window and plotted separately for the three London zones (Fig. 6, top row). The Central-zone profiles generally exhibit higher initial TAD values, with partial overlap with those of the West zone. In contrast, the East-zone profiles generally begin at lower TAD values and show greater variation in their trajectories, including more frequent intersections among profiles. This suggests greater within-zone variation in the scale-dependent behavior of TAD in the East zone, whereas the Central- and West-zone profiles tend to follow more similar and approximately parallel trajectories.

To derive comparable tail slopes across the 192 analysis windows, the head-tail transition was first detected separately for each TAD profile using the automated gradient-matching procedure described above. The most frequently identified transition level was then adopted as the common cutoff for Analysis 2. Accordingly, scale level $s = 5$ was used to separate the head and tail sections for all profiles. The distributions of the coefficients of determination for the fitted tail sections are presented in Supplementary Fig. S3(b).

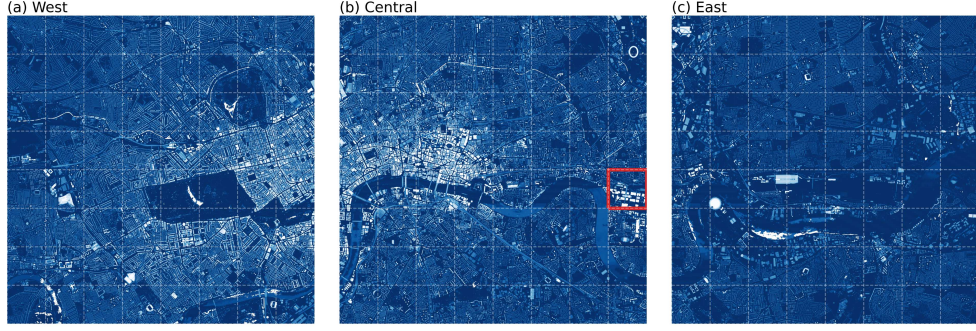


Fig. 5 Normalized Digital Surface Models (nDSMs) for the three selected London zones: (a) West, (b) Central, and (c) East. White dashed lines indicate the boundaries of the $1 \text{ km} \times 1 \text{ km}$ analysis windows, and the red box marks the location of Canary Wharf. The color scale ranges from dark for lower surface heights to bright for higher surface heights. The Central zone contains the most extensive concentration of high-rise structures, whereas vertical development is more dispersed in the West and less prominent in the East.

To facilitate comparison among the TAD profiles, the 192 tail slopes were classified into four equal-sized groups using a quantile-based classification [59] (Fig. 6, bottom row). Each group contains 48 analysis windows. The categories are labeled Highest, Mid-high, Mid-low, and Lowest according to the inferred level of vertical complexity. The Highest category contains the most negative and therefore steepest tail slopes, representing the fastest decrease in TAD across scale levels, whereas the Lowest category contains the least negative and therefore flattest tail slopes, representing the slowest decrease.

Figure 7(a) compares the distributions of the head and tail slopes across the three London zones. The head slopes are generally small in magnitude, with mean values of -0.052 ± 0.037 for the West zone, -0.052 ± 0.032 for the Central zone, and -0.083 ± 0.056 for the East zone. These relatively shallow head slopes indicate that TAD decreases gradually across the initial aggregation levels, as each analysis window is successively subdivided into 4, 16, 64, 256, and finer cells. The West and Central zones exhibit nearly identical mean head slopes, while the East zone has a somewhat more negative mean and greater variability. Nevertheless, the head-slope distributions overlap substantially across the three zones.

Beyond the common cutoff at scale level $s = 5$, the TAD profiles decrease at different rates across the three London zones. A Welch ANOVA identified significant differences in the tail slopes among the West, Central, and East zones ($p < 0.001$, $\eta^2 = 0.23$; Supplementary Table S2). Subsequent Games–Howell post-hoc tests indicated that all three pairwise differences were statistically significant, with effect sizes ranging from moderate to strong (Supplementary Table S3). As shown in Fig. 7(a), the Central zone has the most negative tail slopes on average, indicating the steepest decrease in TAD and therefore the highest average vertical complexity among the three zones. The West zone exhibits less negative slopes than the Central zone, although their distributions overlap substantially. The East zone has the least negative tail slopes on average, indicating a comparatively flatter decrease in TAD. The overlap among

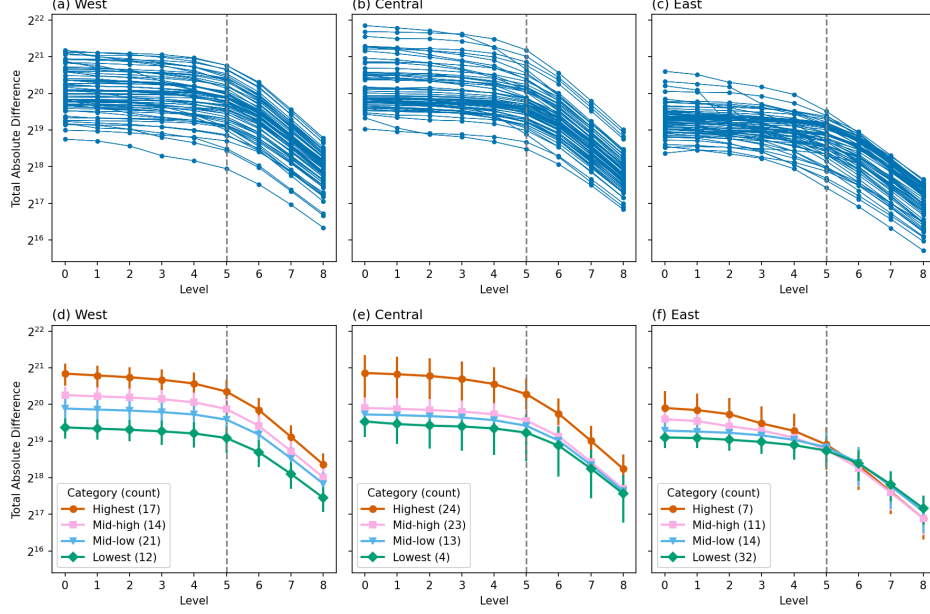


Fig. 6 TAD profiles for the 64 analysis windows in each London zone. The top row shows the individual profiles, while the bottom row shows profiles aggregated by the four tail-slope categories; points represent the mean TAD at each scale level and whiskers indicate ± 1 standard deviation. The differences in profile position and tail behavior reveal variation both between the three zones and among windows within each zone.

the distributions shows that considerable within-zone variation remains despite the statistically significant differences in their central tendencies.

Figure 7(b) further compares the composition of the four vertical-complexity categories across the three zones. The Central zone contains the largest numbers of windows in the Highest and Mid-high categories, with 24 and 23 windows, respectively; 13 windows fall within the Mid-low category, and four within the Lowest category. The West zone has a more balanced distribution, comprising 17 Highest, 14 Mid-high, 21 Mid-low, and 12 Lowest windows. In contrast, the East zone is dominated by the Lowest category, which contains 32 windows, followed by 14 Mid-low, 11 Mid-high, and seven Highest windows. These distributions show that higher-complexity windows are concentrated in the Central zone, while lower-complexity windows are more prevalent in the East zone, with the West zone displaying a more mixed composition.

Figure 8 shows the spatial distribution of the four vertical-complexity categories across the three London zones. In the Central zone (Fig. 8(b)), many windows in the Highest category form an East–West concentration through the central part of the zone, corresponding broadly to London’s central business district (CBD). This concentration extends into the eastern portion of the West zone (Fig. 8(a)), where additional windows are classified within the Mid-high and Mid-low categories. A separate cluster of Highest-category windows occurs along the eastern edge of the Central zone around Canary Wharf and its neighboring areas, where high-rise development is interspersed

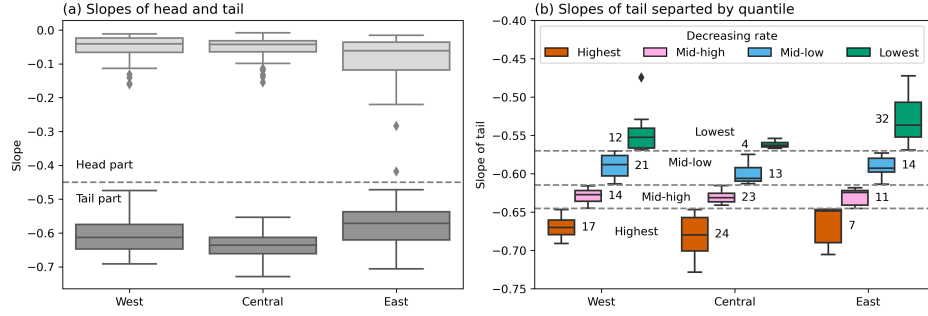


Fig. 7 Distributions of the fitted slopes across the three London zones. (a) Head- and tail-slope distributions by zone. (b) Tail-slope distributions by zone within the four equal-sized vertical-complexity categories: Highest, Mid-high, Mid-low, and Lowest. The number beside each box indicates the number of analysis windows in that zone and category; each category contains 48 windows across the three zones, and the dashed lines show the classification thresholds at -0.645 , -0.615 , and -0.570 . Head slopes overlap substantially across the zones, whereas the tail slopes distinguish the higher average complexity of Central London from the flatter profiles prevalent in the East zone.

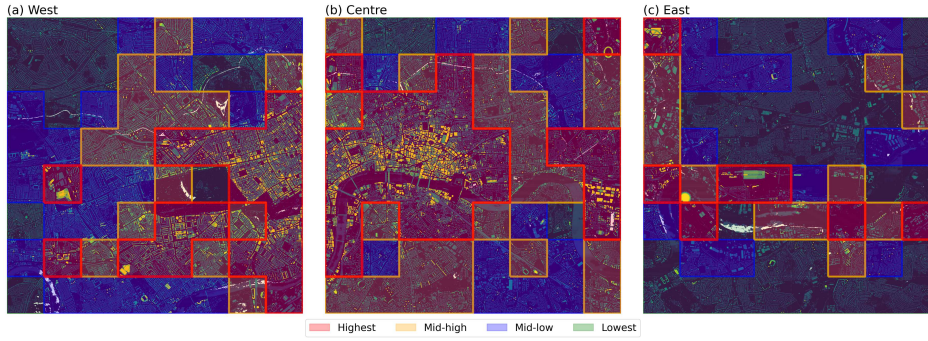


Fig. 8 Spatial distribution of the four vertical-complexity categories across the West, Central, and East London zones. Each $1 \text{ km} \times 1 \text{ km}$ analysis window is classified as Highest, Mid-high, Mid-low, or Lowest according to its tail slope. Higher-complexity windows form concentrations around central London and Canary Wharf, whereas the Lowest category is more prevalent in the East zone.

with lower-rise structures and open spaces. In contrast, most analysis windows in the East zone (Fig. 8(c)) fall within the Mid-low or Lowest categories.

The mapped categories summarize spatial variation in the tail slopes and therefore in the rate at which TAD decreases across finer scale levels. Windows with flatter tail slopes retain relatively similar discrepancies across successive levels and are interpreted as having comparatively simpler vertical structure. Windows with steeper tail slopes exhibit a more rapid reduction in TAD as finer spatial detail is introduced and are interpreted as having greater vertical complexity. These patterns are associated with the organization and heterogeneity of the vertical surface rather than with building height alone.

The progression from the small and nested window analysis in Canary Wharf to the $192 \text{ km} \times 1 \text{ km}$ London zones examines whether the tail slope can differentiate vertical landscapes across a broader metropolitan setting. Figures 6 to 8 show that the

head slopes vary comparatively little across the three zones, whereas the tail slopes differ significantly and exhibit distinct spatial distributions. This divergence indicates that the tail slope can distinguish between the vertically heterogeneous landscapes associated with central London and Canary Wharf and the more uniform vertical structures prevalent in many residential areas. Higher-complexity windows are concentrated around central London, while lower-complexity windows are more prevalent in the East zone. These results support the interpretation that the tail slope captures differences in the scale-dependent organization of vertical urban structure across contrasting urban environments.

Analysis 3: Comparative analysis of 17 urban-zone windows

The third analysis compares vertical complexity across 17 standardized urban-zone windows, comprising three zones in London and 14 city-centred windows elsewhere in England (Supplementary Fig. S1 and Supplementary Table S1). Each urban-zone window has dimensions of $8 \text{ km} \times 8 \text{ km}$, corresponding to an area of 64 km^2 . These standardized windows provide a consistent spatial basis for comparison and are not intended to represent the complete geographic extent of the corresponding cities.

Vertical complexity is examined at two spatial extents. First, a single TAD profile and tail slope are derived for each complete $8 \text{ km} \times 8 \text{ km}$ urban-zone window, enabling comparison of the overall scaling characteristics across the 17 zones. Second, each urban-zone window is subdivided into 64 non-overlapping $1 \text{ km} \times 1 \text{ km}$ analysis windows, producing 1,088 subwindow-level tail slopes in total. Their distributions and spatial patterns are then compared to examine within-zone variation in vertical complexity.

Figure 9(a) displays the TAD profiles for the 17 $8 \text{ km} \times 8 \text{ km}$ urban-zone windows, with four profiles highlighted as examples. The London (Central) profile has the highest initial TAD value. The Manchester profile begins above 2^{25} . York has the lowest initial TAD value among the highlighted examples, beginning slightly above 2^{24} , whereas Southampton occupies an intermediate position. Although the Manchester profile initially lies above the Southampton profile, the two intersect between scale levels $s = 6$ and $s = 7$, indicating that differences in initial TAD value do not necessarily correspond to differences in the rate of decline across the tail section.

Figure 9(b) shows the fitted head and tail sections for the four highlighted urban-zone windows. Using the profile-level transition detection and majority-agreement procedure, scale level $s = 5$ was selected as the common cutoff for the 17 zone-level profiles. Among the four examples, London (Central) has the most negative tail slope and therefore the steepest decrease in TAD, followed by Manchester and York. Southampton has the least negative tail slope and consequently the flattest decrease. The ordering of the tail slopes differs from the ordering of the initial TAD values, reinforcing that the tail slope characterizes the scale-dependent rate of information recovery rather than the overall magnitude of vertical variation alone.

Importantly, the initial value of a TAD profile does not necessarily correspond to its tail slope. For example, the Southampton urban-zone window has a relatively high initial TAD value of approximately 2^{25} , exceeding that of York, which begins near 2^{24} . Nevertheless, Southampton has the least negative tail slope among the highlighted

examples (-0.464), whereas York has a slightly steeper tail slope (-0.478). This contrast indicates that the initial TAD magnitude and the tail slope capture distinct aspects of the vertical landscape: the former reflects the overall discrepancy between coarse and high-resolution representations, while the latter characterizes the rate at which that discrepancy decreases across finer scale levels. Consequently, an urban-zone window with a higher initial TAD value does not necessarily exhibit greater vertical complexity.

The Southampton–York comparison raises a broader question: does the tail slope characterize vertical complexity, or does it primarily reflect the absolute height of the urban landscape? To examine this distinction systematically, we assessed the correlations between the absolute tail slope and four comparison measures of vertical structure: the Gini coefficient of height values, maximum height, the 99th-percentile height, and the 90th-percentile height (Table 2). Maximum and upper-percentile heights characterize the presence and intensity of tall structures, whereas the Gini coefficient characterizes inequality in the distribution of surface heights.

The absolute tail slope is positively associated with the height-based measures at both analysis-window sizes. At the $8 \text{ km} \times 8 \text{ km}$ urban-zone level ($n = 17$), the Pearson correlation with maximum height is moderate and statistically significant ($r = 0.623$, $p = 0.008$), while the correlations with the 99th- and 90th-percentile heights are stronger ($r = 0.765$, $p < 0.001$, and $r = 0.689$, $p = 0.002$, respectively). Similar relationships are observed across the 1,088 $1 \text{ km} \times 1 \text{ km}$ analysis windows, where the absolute tail slope is moderately to strongly associated with maximum height and the upper-percentile heights. The corresponding Spearman correlations show broadly similar patterns. These results indicate that windows containing taller vertical features tend to exhibit steeper tail slopes.

The correlations are nevertheless insufficient to reduce the tail slope to a direct measure of height. At the urban-zone level, even the strongest Pearson relationship, with the 99th-percentile height, accounts for 58.5% of the variation in the absolute tail

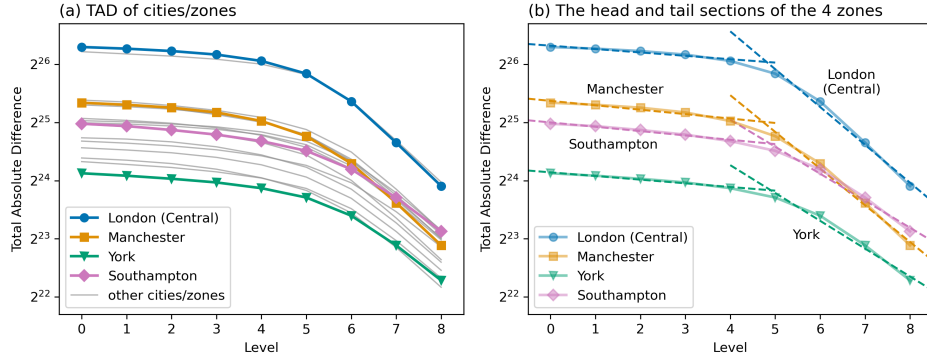


Fig. 9 TAD profiles and fitted scaling sections for the 17 standardized urban-zone windows. (a) TAD profiles across scale levels, with London (Central), Manchester, Southampton, and York highlighted as examples. (b) Fitted head and tail sections for the four highlighted profiles, using scale level $s = 5$ as the common cutoff. The contrasting order of their initial TAD values and tail slopes demonstrates that the magnitude of a TAD profile and its scale-dependent rate of decline represent distinct characteristics of the vertical landscape.

Table 2 Correlations between the absolute tail slope and comparison measures of vertical structure, including the Gini coefficient, maximum height, and the 99th- and 90th-percentile heights. Larger absolute tail slopes indicate steeper tail sections.

Window dimensions	n	Method	Gini coefficient	Max height	99th percentile	90th percentile	
8 km ×8 km	17	Pearson's	r	-0.283	0.623	0.765	0.689
			r^2	0.080	0.388	0.585	0.474
			p-value	0.272	0.008	< 0.001	0.002
		Spearman's	ρ	-0.199	0.574	0.696	0.691
			p-value	0.445	0.016	0.002	0.002
1 km ×1 km	1,088	Pearson's	r	-0.012	0.571	0.641	0.635
			r^2	0.000	0.326	0.411	0.403
			p-value	0.698	< 0.001	< 0.001	< 0.001
		Spearman's	ρ	-0.235	0.640	0.768	0.692
			p-value	< 0.001	< 0.001	< 0.001	< 0.001

slope ($r^2 = 0.585$). The relationship with the Gini coefficient is weak and statistically non-significant at this level ($r = -0.283$, $p = 0.272$). At the 1 km × 1 km level, the linear relationship with the Gini coefficient is negligible ($r = -0.012$, $p = 0.698$), although the weak negative Spearman correlation ($\rho = -0.235$, $p < 0.001$) suggests a limited monotonic association. Thus, the tail slope is related to verticality but is not equivalent to absolute height or to inequality in the height distribution.

These findings clarify the distinction between verticality and vertical complexity. Highly urbanized central areas may be more likely to exhibit steep tail slopes because they often combine high-rise buildings, mid- and low-rise structures, streets, open spaces, and irregular spatial arrangements within the same analysis window. Such mixtures reveal additional vertical variation as progressively finer scale levels are introduced. However, high verticality alone does not necessarily imply high vertical complexity. A regularly arranged district composed of buildings with similar heights may contain tall structures while exhibiting comparatively limited variation across scales. The tail slope should therefore be interpreted as a scaling-based measure of how vertical structure is spatially organized and progressively revealed, rather than as a direct proxy for building height.

Figure 10 summarizes the head and tail slopes derived from the 17 8 km × 8 km urban-zone windows. The distributions of the coefficients of determination for the fitted tail sections are presented in Supplementary Fig. S3(c). The absolute head slopes are consistently shallow across the 17 windows, with a mean of -0.068 ± 0.011 (Fig. 10(a)), indicating that TAD decreases relatively slowly across the initial scale levels. In contrast, the tail slopes show substantially greater variation among the urban-zone windows. As shown by their absolute values in Fig. 10(b), London (Central) has the steepest tail slope, followed by Manchester, London (West), Newcastle, Birmingham, and Liverpool, whereas Southampton has the flattest tail slope. A larger absolute tail slope indicates a more rapid decrease in TAD as progressively finer scale levels are introduced. The variation in these values therefore indicates differences in the scale-dependent organization of vertical structure among the 17 urban-zone windows.

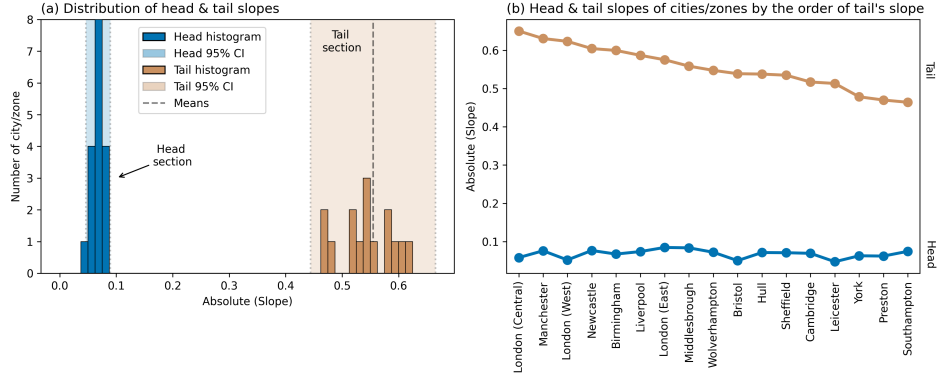


Fig. 10 Head- and tail-slope characteristics of the 17 standardized urban-zone windows. (a) Distributions of the absolute head and tail slopes across the 17 windows, with shaded bands showing their between-window dispersion. (b) Absolute head and tail slopes for each $8 \text{ km} \times 8 \text{ km}$ window, ordered by absolute tail slope. The narrow distribution of head slopes and wider variation in tail slopes show that differences among the urban-zone windows emerge primarily in the finer-scale tail regime.

Across the three analyses, the TAD profiles generally exhibit a shallow head section followed by a steeper tail section. The relatively consistent head slopes indicate that TAD changes slowly across the initial, coarser aggregation levels, whereas the greater variation in the tail slopes emerges as progressively finer spatial detail is introduced. The tail slope therefore provides the principal basis for comparing the scale-dependent vertical complexity of the analysis windows.

At the urban-zone level, windows with similar absolute tail slopes exhibit comparable rates of TAD reduction, although this does not imply that their underlying urban forms are identical. For example, Manchester and London (West) have similar tail slopes, while Bristol, Hull, and Sheffield occupy a similar intermediate range. These groupings provide an empirical comparison of vertical-complexity levels across the sampled urban-zone windows, while the distinct spatial configurations responsible for those slopes require interpretation alongside the corresponding nDSM data and finer-scale results.

To examine within-zone variation at a finer spatial extent, Fig. 11 presents the distributions of absolute tail slopes for the $64 \text{ } 1 \text{ km} \times 1 \text{ km}$ analysis windows within each of the 17 urban-zone windows. The distributions of the coefficients of determination for the fitted tail sections are shown in Supplementary Fig. S3(d). Across the 1,088 analysis windows, most absolute tail slopes fall approximately between 0.15 and 0.80. Within the present dataset, windows with values below approximately 0.30, including several in York and Southampton, tend to exhibit comparatively flatter tail sections, whereas windows with values above approximately 0.60, including many in London (Central) and parts of Manchester, exhibit steeper tail sections. These values should be interpreted as empirical reference ranges for the sampled windows rather than as universal thresholds of urban vertical complexity.

The distributions also reveal substantial variation both between and within urban zones. London (Central) contains a relatively concentrated distribution at the higher end of the observed range, whereas several other urban zones exhibit wider spreads

across low, intermediate, and high absolute tail slopes. This within-zone variation shows that a single zone-level tail slope summarizes only the overall scaling behavior of an $8 \text{ km} \times 8 \text{ km}$ window, while the $1 \text{ km} \times 1 \text{ km}$ results reveal finer spatial heterogeneity within that extent.

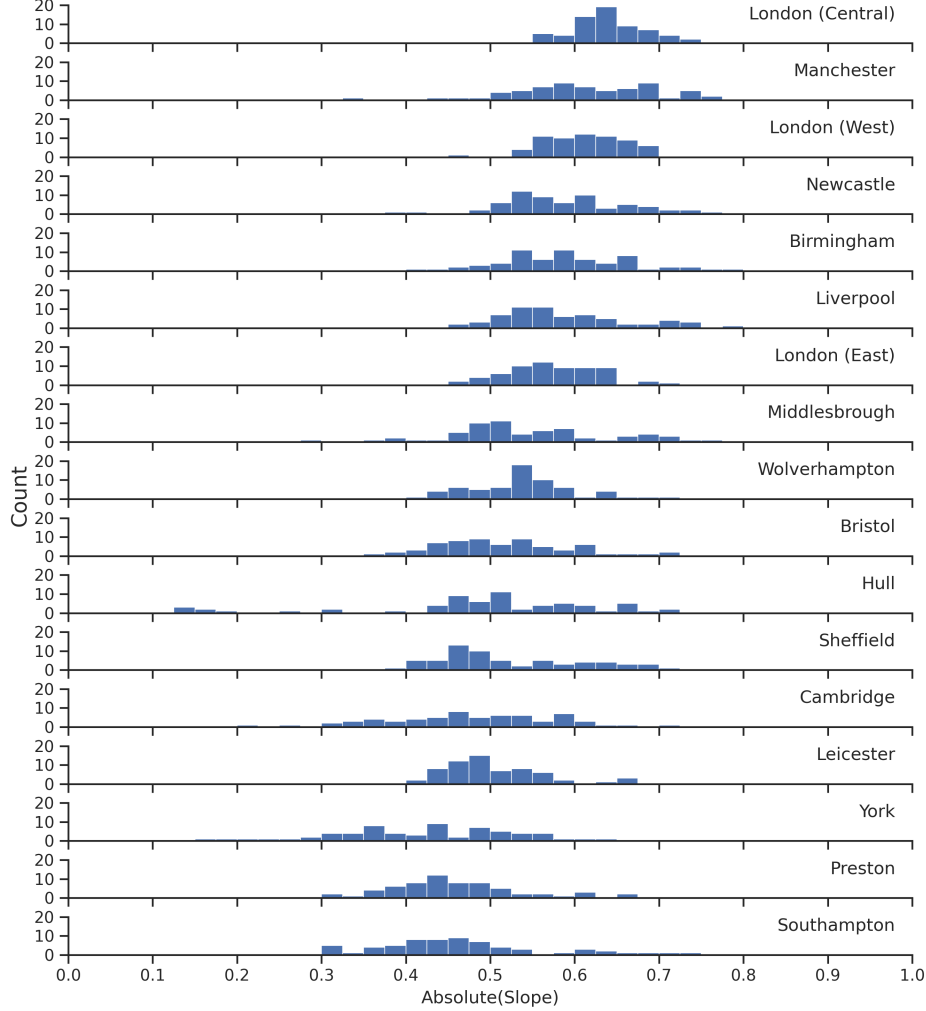


Fig. 11 Distributions of absolute tail slopes for the 64 $1 \text{ km} \times 1 \text{ km}$ analysis windows within each of the 17 standardized urban-zone windows. Each distribution summarizes within-zone variation at the subwindow level. The results reveal both systematic differences between urban zones and substantial spatial heterogeneity within individual zones, which is not captured by a single zone-level tail slope.

The subwindow-level correlations in [Table 2](#) provide further context for these distributions. Across the 1,088 $1 \text{ km} \times 1 \text{ km}$ windows, the absolute tail slope is moderately

associated with maximum height ($r = 0.571$, $p < 0.001$) and more strongly associated with the 99th- and 90th-percentile heights ($r = 0.641$ and $r = 0.635$, respectively; both $p < 0.001$). The corresponding Spearman correlations are also positive and statistically significant, particularly for the 99th-percentile height ($\rho = 0.768$, $p < 0.001$). These relationships indicate that subwindows containing taller vertical features generally tend to exhibit steeper tail slopes. However, the associations remain incomplete, and the linear relationship with the Gini coefficient is negligible ($r = -0.012$, $p = 0.698$), although a weak negative monotonic relationship is detected by Spearman correlation ($\rho = -0.235$, $p < 0.001$). The subwindow-level results therefore reinforce the interpretation that the tail slope is related to verticality but also reflects aspects of vertical organization not captured by height magnitude or height inequality alone.

The comparison across 1,088 subwindows extends the nested-window findings from analysis 1 to a broader set of urban environments. In relatively uniform vertical landscapes, the TAD profile tends to decrease more gradually as finer scale levels are introduced, resulting in flatter tail slopes. In more heterogeneous landscapes, additional vertical variation is revealed across successive levels, producing steeper tail slopes. The subwindow distributions therefore characterize spatial variation within the standardized urban-zone windows rather than defining a single complexity signature for an entire city.

Discussion

In this study, we used nDSM raster data to introduce a framework for analysing the scale-dependent organization of urban vertical structure. Building on the quadtree data structure, the framework generates progressively finer representations of a fixed analysis window by calculating the average surface height within cells at successive scale levels. Each aggregated representation is compared with the original high-resolution surface using the Total Absolute Difference (TAD). The sequence of TAD values forms a TAD profile, from which the tail slope is derived as the principal metric of urban vertical complexity.

For a relatively uniform vertical landscape, progressively finer subdivision reveals comparatively little additional variation, and the TAD decreases more gradually. In a more heterogeneous landscape, finer scale levels progressively resolve additional contrasts among surface features, resulting in a steeper tail slope. The tail slope therefore characterizes the rate at which vertical variation is revealed across scale levels, rather than the TAD magnitude at any single level.

Across the analysis windows examined in this study, the TAD profiles generally exhibit a two-regime decreasing pattern: a relatively shallow head section at coarser scale levels followed by a steeper tail section at finer levels. The head slopes occupy a comparatively narrow range, whereas the tail slopes vary more substantially among analysis windows. This indicates that differences in vertical organization emerge primarily after the representation becomes sufficiently fine to resolve localized surface variation.

The transition between the two regimes provides an empirical turning point within each TAD profile. It may be interpreted as the approximate scale level at which the

representation shifts from capturing broad contrasts across the analysis window to increasingly resolving localized vertical structure. Canary Wharf provides a visual illustration of this transition (Fig. 2). At scale levels 0 to 3, the representation remains coarse: the main visible pattern is that the southern portion of the window is generally higher than the northern portion, while individual building forms are not clearly resolved. At scale levels 4 and 5, the principal spatial arrangement of the vertical surface and recognizable building outlines begin to emerge. Further subdivision at scale levels 6 to 8 adds progressively finer detail, but the broad spatial configuration remains similar to that already visible around the turning point. The turning point may therefore identify a level at which the principal vertical pattern becomes recognizable, while the subsequent tail section mainly refines localized outlines and height contrasts. This interpretation is based on visual observation of the Canary Wharf example and should not be treated as a universal physical or perceptual threshold.

The common cutoff was estimated separately between the three analyses because the profile sets and the dimensions of analysis-window differed. Accordingly, the cutoff at $s = 4$ in Analysis 1 and at $s = 5$ in Analyses 2 and 3 are not contradictory. Each cutoff provides a consistent basis for fitting and comparing tail slopes within the corresponding analysis, rather than defining one universal transition level for all window dimensions.

The three analyses provide complementary views of vertical complexity at different observation extents. This distinction is important because the tail slope is calculated within a defined analysis window, so its interpretation depends on the spatial extent over which vertical variation is aggregated. Together, the nested Canary Wharf analysis, the London intra-urban comparison, and the 17-zone comparison show that the tail slope should be interpreted as a window-specific summary of scale-dependent vertical organization rather than as a universal property of a city or location. Window size is therefore not merely a technical setting, but part of the substantive interpretation of vertical complexity: larger windows summarize broader urban-zone structure, whereas smaller windows retain localized contrasts among buildings, streets, open spaces, and other surface elements.

The results further clarify the relationship between verticality and vertical complexity. The absolute tail slope is positively associated with maximum height and upper height percentiles at both the $8 \text{ km} \times 8 \text{ km}$ and $1 \text{ km} \times 1 \text{ km}$ window sizes. Analysis windows containing taller surface features therefore tend, on average, to exhibit steeper tail slopes. Nevertheless, these relationships account for only part of the observed variation, while the associations with the Gini coefficient of surface heights are weak or negligible. The tail slope is therefore related to verticality but is not equivalent to absolute height or height inequality.

The contrast between initial TAD magnitude and tail slope reinforces this distinction. Initial TAD reflects the overall discrepancy between a coarse representation and the original surface, whereas the tail slope describes how rapidly that discrepancy decreases through finer scale levels. Southampton, for example, has a higher initial TAD value than York but a flatter tail slope. A higher initial discrepancy therefore does not necessarily indicate greater vertical complexity.

Urban vertical complexity should consequently be interpreted as the scale-dependent organization of vertical variation rather than the presence of tall buildings alone. High-rise central areas may frequently exhibit steep tail slopes because they often combine towers, mid- and low-rise buildings, streets, open spaces, vegetation, and irregular spatial arrangements within the same window. However, a regularly arranged district composed of buildings with similar heights and spacing could exhibit high verticality but comparatively limited vertical complexity. Conversely, a lower-rise environment with diverse heights and irregular spatial arrangements may produce a relatively steep tail slope despite having a lower maximum height.

This distinction is also important when interpreting the mapped slope categories. The Highest category does not simply identify the tallest windows. It identifies windows in which TAD decreases most rapidly through the tail section. The resulting patterns therefore reflect both the vertical characteristics of the surface and how those characteristics are spatially organized within each analysis window.

The recurrence of a shallow head and a steeper tail across the three analyses suggests that the two-regime TAD profile is a consistent empirical feature of the present dataset. It should not, however, be interpreted as a universal scaling law. The study areas share a national context, a common data source, and related analytical settings. Urban environments with different planning histories, terrain, building traditions, vegetation patterns, or data quality may exhibit different profile shapes, turning points, and slope distributions. The framework is therefore proposed as a comparative approach rather than a singular universal model of urban vertical form.

The application to lower-rise urban zones such as York, Cambridge, and Wolverhampton further shows that the framework is not restricted to prominent high-rise centres. Their flatter tail slopes indicate that finer subdivision generally reveals less additional vertical variation than in more heterogeneous high-rise environments. At the same time, the subwindow distributions demonstrate that lower-rise urban zones are not necessarily uniform; localized landmarks, mixed building types, vegetation, and irregular development can still generate substantial within-zone variation.

The observed slope ranges and category boundaries should therefore be treated as empirical descriptions of the current sample rather than universal thresholds. Values associated with comparatively flatter or steeper profiles under the present settings may shift when the framework is applied to different window dimensions, data resolutions, urban forms, or geographic contexts.

While this study treats urban vertical structure as a static snapshot, the framework could also be applied to multi-temporal elevation datasets to examine changes in vertical organization through construction, redevelopment, demolition, or vegetation change. A temporal shift toward a steeper or flatter tail slope would indicate a change in scale-dependent vertical structure. It should not, however, be attributed automatically to a particular planning intervention or socio-economic process without independent supporting evidence.

The tail slope may also provide a structural variable for future research on the socio-economic and planning processes associated with urban form. Differences in land values, development histories, regulations, and planning practices may plausibly contribute to variation in vertical morphology, but these mechanisms were not tested

directly in this study. The present results therefore do not establish that the observed slopes represent economic agglomeration, urban self-organization, planning quality, urban vibrancy, environmental performance, or the success of densification policies. Evaluating such relationships would require independent explanatory variables and appropriate comparative or longitudinal research designs.

The turning point may similarly have potential perceptual relevance because it marks the approximate level at which broad vertical patterns begin to resolve into more recognizable local structures. This could relate to the spatial grain at which urban form becomes visually legible [60]. However, the present analysis includes no perceptual or behavioural observations. Any relationship between the turning point, urban legibility, and human experience should therefore be regarded as a hypothesis for future research rather than a demonstrated implication of the metric.

Several recommendations for future studies can be made to further test and strengthen the proposed concept and framework. First, although the case studies in this paper focus on urban zones in England due to data availability and cost considerations, the same analysis framework could be applied to other geographic contexts, particularly cities in East Asia and Southeast Asia, where dense urban lifestyles are more vertically designed, integrated, and developed [1]. While we utilize high-resolution LiDAR-derived datasets, the emergence of global Level of Detail 1 (LoD-1) building datasets offers a potential pathway for scaling this analysis globally [61]. Future research could investigate how the abstraction of buildings into LoD-1 polygons—and the associated model uncertainties—affects the stability of the TAD scaling signature compared with direct physical DSM measurements. Because the present nDSM also includes vegetation and other above-ground, non-building objects, comparison with building-only height models would further help determine how different surface components contribute to the measured vertical complexity.

Second, if raster data from different time frames for the same area become available, it would be possible to measure and explore dynamic changes in urban vertical complexity during urban development and redevelopment processes. Analyzing these changes could provide additional insights into the temporal evolution of urban morphology. However, a change in tail slope would indicate a change in scale-dependent vertical organization and should not automatically be attributed to changes in urban vibrancy, planning intervention, or other socio-economic processes without independent supporting evidence.

Third, the framework has so far been evaluated using a limited set of analysis-window dimensions and one fixed grid alignment. The standardized $8\text{ km} \times 8\text{ km}$ windows provide a consistent basis for comparison but do not represent the complete geographic or morphological extent of the corresponding cities. Similarly, the $1\text{ km} \times 1\text{ km}$ values are window-specific summaries rather than estimates of a continuous local surface. Neighboring windows may also share broader spatial influences and may not be statistically independent, although spatial dependence among adjacent tail slopes was not explicitly modeled in this study.

Future research could evaluate the sensitivity of the tail slope to alternative analysis-window sizes, raster resolutions, and window alignments. Changing the window size would alter the spatial extent summarized by the metric, whereas shifting or

overlapping the windows could reveal sensitivity to the placement of their boundaries. Such tests would provide a more systematic assessment of aggregation and zoning effects associated with the MAUP. The fixed grid alignment used in this study should therefore be regarded as a technical choice rather than evidence that the resulting tail slopes are invariant to window placement.

The estimated turning points, common cutoffs, and observed tail-slope ranges should also be interpreted as empirical results specific to the present datasets and analytical settings. Their values may vary with analysis-window dimensions, source-data resolution, and urban morphology. Accordingly, the slope ranges and category boundaries reported in this study should not be interpreted as universal thresholds separating lower- and higher-complexity urban environments.

Fourth, we opted for absolute differences (TAD) over squared-difference metrics to reduce the disproportionate influence of extreme values while retaining an interpretable measure of total discrepancy. Future iterations of this framework could explore the sensitivity of the scaling profiles to alternative metrics, such as squared-difference measures or root-mean-square error. Such research would help determine whether these metrics offer additional nuances in capturing vertical complexity or whether they are disproportionately influenced by urban outliers such as high-rise clusters.

The correlation analysis indicates that the tail slope is related to conventional measures of verticality, particularly maximum height and upper height percentiles. It should therefore not be interpreted as completely independent of surface height. Instead, the tail slope reflects how vertical variation is spatially organized and progressively resolved across scale levels. Future validation using controlled synthetic landscapes or urban areas with similar height distributions but different spatial arrangements could help distinguish the influence of height magnitude from that of morphological organization.

Lastly, another direction for future studies is to investigate the relationship between urban vertical structure, demographic and socio-economic characteristics, planning policies, environmental conditions, human activity, and perception. While this study visually examined the relationship between building and street outlines within selected analysis windows for interpretive purposes, these relationships were not tested quantitatively. A more comprehensive analysis using independent explanatory variables could yield a deeper understanding of how vertical complexity relates to human activities and urbanization processes. Such applications would require appropriate comparative, longitudinal, or causal research designs and should not infer planning quality, urban vibrancy, or socio-economic outcomes from the tail slope alone.

In summary, this study introduces a scaling-based framework for characterizing urban vertical complexity from high-resolution elevation data. The framework generates a TAD profile by comparing progressively finer aggregated representations with the original nDSM surface and uses the tail slope as the principal comparative metric. Across the nested-window analysis in Canary Wharf, the intra-urban analysis across London, and the comparison of 17 standardized urban-zone windows, the tail slope differentiates analysis windows according to the rate at which vertical variation is progressively resolved across scale levels. The results also show that the tail slope is

related to conventional measures of verticality, particularly upper height percentiles, but is not equivalent to absolute height, initial TAD magnitude, or inequality in the height distribution.

The proposed metric should therefore be interpreted as a window-specific comparative descriptor of scale-dependent vertical organization rather than as a universal urbanization index, a complete representation of an entire city, or a direct indicator of planning quality. Its interpretation depends on the dimensions, composition, alignment, and source resolution of the analysis window. Further evaluation across broader geographic contexts, alternative spatial-unit configurations, different surface representations, and multi-temporal datasets is needed to establish the stability, transferability, and potential applications of the observed scaling behavior.

Methods

This study proposes a framework for measuring and quantifying urban vertical complexity. In the following sections, we first describe the study areas and datasets. We then present the framework sequentially: (1) the spatial scaling framework, (2) the calculation of Total Absolute Difference (TAD) profiles, and (3) the identification of the turning point and tail slope. Finally, we describe the experimental design and statistical analyses used to evaluate the proposed framework.

Study areas and datasets

The case study utilizes datasets from 17 standardized urban-zone windows in England, including three zones in London (West, Central, and East) and 14 city-centred windows in Birmingham, Manchester, Newcastle, Sheffield, Bristol, Liverpool, Leicester, Middlesbrough, Southampton, Preston, Hull, Wolverhampton, York, and Cambridge (Supplementary Table S1). The datasets for the analysis include the DSM and DTM raster data [62, 63]. The DSM and DTM data were generated from airborne LiDAR, which respectively contained the height values of the natural or built environment surfaces and the height values of the ground (from sea level) for every two-dimensional cell of dimensions $2\text{ m} \times 2\text{ m}$. The raw data is organized in $1\text{ km} \times 1\text{ km}$ raster tiles, each containing a matrix $\mathbf{D}$ of 500×500 height values. Each standardized urban-zone window has dimensions of $8\text{ km} \times 8\text{ km}$ and comprises 64 source raster tiles. These windows provide a consistent spatial basis for comparison and are not intended to represent the complete geographic or morphological extent of the corresponding cities.

The normalized DSM (nDSM) data is generated based on the DSM and DTM data. This data preprocessing procedure consists of two steps. To ensure surface continuity for the scaling analysis, missing value cells (representing less than 10% of the total dataset) were filled using inverse distance weighting (search radius: 200 m, inverse distance power: 2) interpolation. This step is necessary to avoid artificial ‘vertical spikes’ or discontinuities that would occur if missing values were left empty. Visual verification of the resulting surfaces (see Fig. 5 and Supplementary Fig. S1) indicates that this process maintains the integrity of the urban texture and does not introduce significant smoothing artifacts that would bias the complexity metrics. Then, the nDSM is calculated by subtracting the ground heights (DTM) from the surface heights

(DSM). The resulting nDSM data contains only the heights of objects on the Earth’s surface, thus representing above-ground surface heights within the analysis window, including both natural objects (trees) and artificial structures (buildings, bus stops, vehicles, etc.). [Figure 5](#) presents the nDSM of three selected zones within London (see Supplementary Fig. S1 for the other 14 urban-zone windows). As expected, most of the high-rise buildings are found within the Central zone, including the City of London and Canary Wharf; the city core extends to the West zone, which contains Westminster, Mayfair, and Belgravia. The East zone contains fewer high-rise buildings compared to the other two zones. All analyses reported in this work are based on nDSM data.

The spatial scaling framework

The first step in the proposed framework is the spatial scaling framework, which involves calculating average surface heights at successive scale levels. This process recursively divides the analysis window into four quadrants of equal size ([Fig. 12](#)). The quadtree data structure is at the core of implementing this spatial scaling framework [[15, 64](#)]. It starts with the entire analysis window as the initial box (scale level 0, the root node). Then, this initial box is split into $2 \text{ columns} \times 2 \text{ rows} = 4$ equal-sized boxes (scale level 1, the first level branches/child nodes). Subsequently, each of the four boxes is again split into four equal-sized boxes, resulting in a total of $2^2 \times 2^2 = 16$ boxes (scale level 2). The splitting process is repeated a total of 8 times to reach the scale level 8 (the finest available scale level of branches/child nodes), thereby generating a total of $2^8 \times 2^8 = 65,536$ boxes. At scale level 8, each box covers an area slightly larger than $2 \text{ m} \times 2 \text{ m}$. In other words, the spatial scaling framework is an iterative procedure that doubles the number of boxes along each axis and halves the box side length at each scale level. We use a large-to-small recursive subdivision starting from a fixed analysis window. Across all three analyses, scale level $s = 8$ corresponds to a box side length of approximately 3.91 m. This maintains a constant spatial extent within each TAD profile and avoids remainder rows or columns that may arise when aggregating upward from the original raster (small-to-large aggregation). The resulting scale levels are therefore internally consistent within each analysis window. Comparisons between windows nevertheless remain conditional on their size, location, and alignment.

The scale level equals the number of times the boxes are split. Each split divides the box’s side length (l) into two equal parts ($l/2$). Therefore, the side length of the box after splitting (l_s) is equal to the division of the total side length (e.g., $L = 1 \text{ km}$ by 2 raised to the power of the scale level (s)). After rearranging the equation, the scale level equals the negative logarithm of the box length ratio (l_s/L , see [Eq. \(1\)](#)). In other words, the linear increment of the scale level corresponds to a logarithmic decrease in the box length ratio, thereby reflecting the hierarchical nature of the spatial scaling framework.

$$\begin{aligned} l_s &= \frac{L}{2^s}, \\ s &= -\log_2 \frac{l_s}{L}. \end{aligned} \tag{1}$$

In the spatial scaling framework, the average height value of the corresponding area is calculated for each box (h_i^s) at a given scale level, see [Eq. \(2\)](#). [Figure 2](#) visually

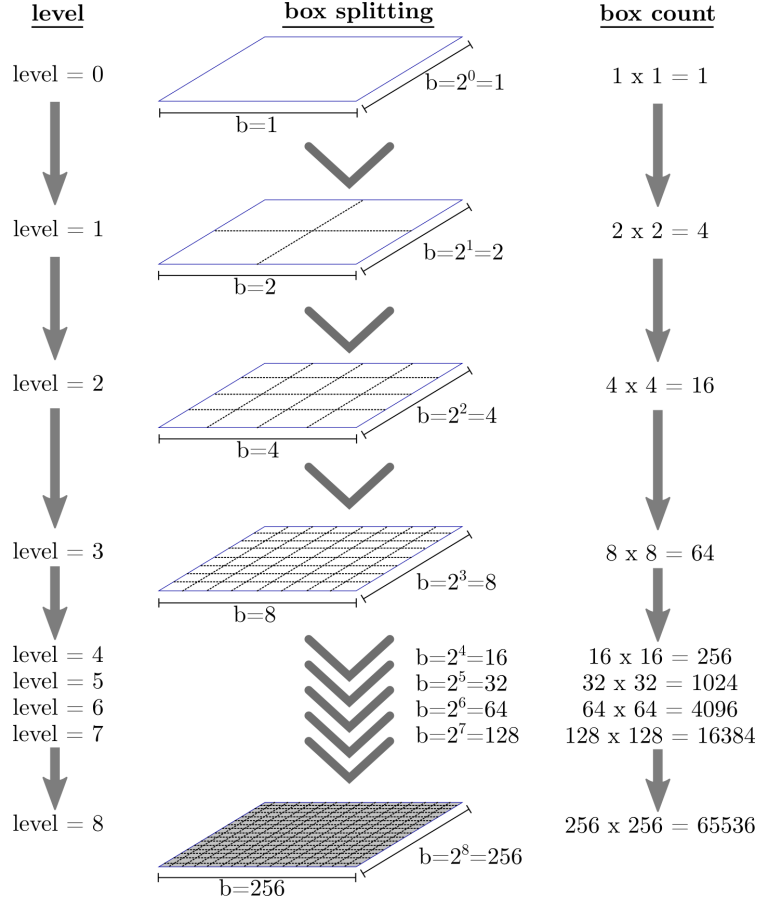


Fig. 12 Schematic representation of the recursive box-splitting and height-aggregation procedure. Each scale level halves the box side length along both spatial dimensions and calculates the average surface height within each resulting box, producing progressively finer representations of the same fixed analysis window.

depicts how the average height responds to the spatial scaling framework for a $1 \text{ km} \times 1 \text{ km}$ analysis window containing Canary Wharf (data id: TQ3780.2M)—the financial district in southern London. There is only one box at scale level 0, and all cell heights are averaged into a single value. At scale level 1, the 250×250 cells at each corner (northeast, northwest, southeast, and southwest) are averaged into four values. A similar average height calculation procedure is applied to the boxes at all scale levels. An area-weighted average function is used when the box splitting intersects raster cells at the box edges ($w_{i,j}$). Technically, this average height calculation process is implemented using a customized quadtree data structure in Python. In this structure,

each node contains an average height value calculated based on the input data.

$$\begin{aligned} h_i^s &= \frac{\sum_{j \in \mathbf{D}} w_{i,j} \times h_j}{\sum_{j \in \mathbf{D}} w_{i,j}}, \\ w_{i,j} &= \frac{\text{area of Cell}_j \text{ in Box}_i}{\text{total area of Cell}_j}. \end{aligned} \quad (2)$$

Scaling of the Total Absolute Differences

Given a static matrix (i.e., a raster image) of the surface elevation (nDSM), if the heights are aggregated into fewer boxes (e.g., 1 box or 4 boxes), the resulting image would be coarse, and the original pattern would be difficult to recognize. Mathematically, this translates into the fact that the discrepancy or difference between the actual heights (at original resolution) and the average heights (aggregated box) for the same locations would be quite large. At progressively finer scale levels, e.g., by adding more boxes (128×128 boxes or 256×256 boxes), the increasing number of boxes produces an aggregated surface that more closely approximates the original nDSM—the corresponding average heights will result in a sharper image of the surface that is closer to the original patterns. Unsurprisingly, the discrepancy is expected to decrease as the scale level increases. As the box dimensions approach the native raster-cell size, the aggregated representation approaches the original nDSM and the discrepancy tends toward zero. If each box corresponds exactly to one original raster cell, the discrepancy is zero.

Conventional three-dimensional fractal analyses commonly use cube-counting approaches, in which the object is partitioned using geometrically comparable units along the horizontal and vertical dimensions. This representation is less suited to the present raster-based analysis because urban surfaces are strongly anisotropic: their horizontal extents span kilometres, whereas their vertical ranges are considerably smaller. The proposed framework therefore retains the nDSM surface representation and measures the discrepancy produced by horizontal spatial aggregation. TAD provides a direct measure of this discrepancy without requiring the horizontal and vertical dimensions to be represented by isotropic three-dimensional cubes.

The Total Absolute Difference (TAD) quantifies the total discrepancy between an aggregated representation at a given scale level and the original high-resolution nDSM surface (Supplementary Fig. S2). As expressed in Eq. (3), the TAD is the weighted sum of the absolute heights differences between two layers: the input data layer (h_j , extracted from matrix $\mathbf{D}$) and the spatial scaling layer (h_i^s).

$$\text{TAD}(s) = \sum_{i \in \text{Box}^s} \sum_{j \in \mathbf{D}} w_{i,j} \times |h_j - h_i^s|, \quad (3)$$

where, h_i^s is the average height of a box i belonging to the set Box^s of boxes at scale level s ; h_j is the original nDSM height of cell j belonging to the matrix $\mathbf{D}$; $w_{i,j}$ is the proportion of the partial area of cell j that intersects with box i over the total area of cell j . A weighted sum equation is used because the cells are not always fully contained within a single box; cells on the borders of boxes may be split into multiple (2 to 4) parts that are located in different boxes.

The difference between actual and average heights is expected to be large at lower scale levels (i.e., lower resolution and fewer boxes). As expected, the TAD decreases when the spatial scaling framework progresses from low to high scale levels. A larger TAD indicates a greater discrepancy between the aggregated representation and the original nDSM surface. In contrast, a smaller TAD indicates a smaller discrepancy and a closer approximation of the aggregated box heights to the original nDSM cell heights. In other words, a lower TAD signifies that more details of the spatial patterns in the input data have been captured and integrated.

To some extent, the TAD reflects the homogeneity in city design. If the entire downtown area consists of high-rise buildings, the resulting TAD of a mid-scale box—covering the building sizes—would be low. Similarly, in the city’s peripheral areas, where there are large expanses of uniformly flat land, the TAD would also be low. At the edge between the downtown area and the peripheral area, the TAD would be high, as the variance in height is maximized. To avoid ambiguity between the spatial and scaling concepts used throughout the manuscript, [Table 3](#) summarizes the key terminology adopted in this study.

Table 3 Key terminology used in the proposed vertical-complexity framework.

Term	Definition	Role in study
Analysis window	The spatial extent over which a TAD profile is computed (e.g., 125 m $\times$ 125 m, 1 km $\times$ 1 km, or 8 km $\times$ 8 km).	Spatial unit of analysis
Base 0–3	A hierarchy of nested analysis windows used in Analysis 1, where Base 0 corresponds to a 1 km $\times$ 1 km window and Base 3 corresponds to a 125 m $\times$ 125 m window.	Window designations
Scale level	A quadtree aggregation level within an analysis window. Higher scale levels correspond to finer spatial detail.	Internal scaling hierarchy
TAD	Total Absolute Difference calculated at a given scale level.	Intermediate measure
TAD profile	The sequence of TAD values across scale levels within an analysis window.	Scaling signature
Turning point	Transition between the head and tail sections of the TAD profile.	Auxiliary descriptor
Tail slope	Slope of the tail section of the TAD profile after the turning point.	Primary vertical-complexity metric

Identification of the turning point and tail slope

When analyzing urban vertical complexity, the key indicator is the rate at which the TAD decreases as scale levels increase. The sequence of TAD values across scale levels forms a TAD profile. The principal metric used to characterize urban vertical complexity is the slope of the tail section of this profile, referred to as the tail slope. TAD therefore represents the discrepancy at an individual scale level, whereas the tail slope characterizes the rate at which that discrepancy decreases across the finer scale levels.

For a relatively uniform vertical landscape, progressively finer subdivision reveals comparatively little additional variation, producing a relatively shallow tail slope. In a more heterogeneous vertical landscape, finer scale levels reveal additional contrasts among buildings, open spaces, vegetation, and other surface features, producing a steeper tail slope. Height alone does not determine this behavior; the tail slope reflects how vertical variation is spatially organized and progressively resolved across scale levels.

The transition between the initial, relatively shallow head section and the later, steeper tail section was first identified separately for each TAD profile using an automated gradient-matching procedure based on first-order polynomial fits. This procedure determines the scale level at which the local rate of change becomes more consistent with the later tail section than with the initial head section. To provide a consistent basis for comparison among analysis windows within the same analysis, the most frequently identified transition level was then adopted as the common cutoff. This procedure resulted in a cutoff at scale level $s = 4$ for Analysis 1 and at $s = 5$ for Analyses 2 and 3.

Linear least-squares regression was applied separately to the head and tail sections of each log-transformed TAD profile. The fitted coefficient of the tail section is reported as the tail slope. Because the slopes are negative, more negative values indicate a faster decrease in TAD and, consequently, greater scale-dependent vertical complexity. Absolute tail slopes are used in figures and correlation analyses where larger values are intended to represent steeper tail sections. The coefficient of determination, R^2 , was calculated to assess the goodness of fit of each fitted section.

Experimental design

The proposed framework was evaluated through three complementary analyses conducted at different observation extents. These analyses were designed to examine whether the tail slope can characterize vertical complexity within nested windows, across an urban region, and among standardized urban-zone windows, while also distinguishing broader between-zone patterns from within-zone variation.

Analysis 1 examines nested analysis windows within Canary Wharf. A $1 \text{ km} \times 1 \text{ km}$ window, designated Base 0, was recursively subdivided into four $500 \text{ m} \times 500 \text{ m}$ windows (Base 1), 16 $250 \text{ m} \times 250 \text{ m}$ windows (Base 2), and 64 $125 \text{ m} \times 125 \text{ m}$ windows (Base 3). A TAD profile and corresponding tail slope were calculated independently for each window. This analysis examines whether the broad contrast between the predominantly low-rise northern portion and the more vertically heterogeneous southern portion remains observable across nested windows of different sizes. Because the smaller base configurations begin at progressively finer scale levels, the analysis is not intended to compare head slopes or turning-point positions directly across Base 0 to Base 3.

Analysis 2 examines intra-urban variation across three London zones: West, Central, and East. Each zone contains 64 non-overlapping $1 \text{ km} \times 1 \text{ km}$ analysis windows, producing 192 windows in total. A TAD profile and tail slope were calculated for each window, and the resulting slopes were compared statistically and spatially across the three zones. The $1 \text{ km} \times 1 \text{ km}$ window size provides a consistent basis for comparison

but is not assumed to represent an intrinsic or optimal local resolution of the metric. Each value is interpreted as a window-specific summary rather than as an estimate of a continuous surface.

Analysis 3 compares vertical complexity across 17 standardized urban-zone windows in England. Each urban-zone window has dimensions of $8 \text{ km} \times 8 \text{ km}$. First, a single TAD profile and tail slope were calculated for each complete urban-zone window to compare broader scaling characteristics among the 17 zones. Second, each urban-zone window was subdivided into 64 non-overlapping $1 \text{ km} \times 1 \text{ km}$ subwindows, producing 1,088 subwindow-level tail slopes. These subwindows were used to examine within-zone variation that is not captured by a single slope calculated for the complete $8 \text{ km} \times 8 \text{ km}$ window. The standardized windows provide a consistent basis for comparison and are not intended to represent the complete geographic or morphological extent of the corresponding cities.

The analysis windows were aligned with the native nDSM grid to preserve the original raster structure and to ensure consistent quadtree subdivision. This fixed alignment is primarily a technical choice and does not, by itself, demonstrate that the resulting tail slopes are invariant to window placement. Shifting or overlapping an analysis window may alter its morphological composition and therefore its TAD profile and tail slope. The present study does not systematically test this sensitivity. Consequently, the reported values should be interpreted as window-specific summaries obtained under a consistent grid alignment. Future work should examine alternative window origins, overlapping windows, and moving-window designs to quantify possible zoning effects and assess the stability of the metric.

Statistical analyses

Differences in tail slopes among the West, Central, and East London zones were evaluated using Welch’s analysis of variance, which does not assume equal variances among groups. Where the overall test was statistically significant, Games–Howell post-hoc comparisons were used to assess pairwise differences between zones. Effect size was reported using η^2 . The detailed results of these tests are reported in the Supplementary Information.

Associations between the absolute tail slope and four comparison measures of vertical structure—the Gini coefficient of surface heights, maximum height, 99th-percentile height, and 90th-percentile height—were evaluated at both the $8 \text{ km} \times 8 \text{ km}$ and $1 \text{ km} \times 1 \text{ km}$ analysis-window levels. Pearson’s product–moment correlation coefficient, r , was used to assess linear relationships, while Spearman’s rank correlation coefficient, ρ , was used to assess monotonic relationships without assuming linearity.

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Competing Interests

The authors declare no competing financial or non-financial interests.

Data availability

All data used in this research are available under an open data license and can be accessed at the following URL: <https://data.gov.uk/dataset/fba12e80-519f-4be2-806f-41be9e26ab96/lidar-composite-dsm-2017-2m> and <https://data.gov.uk/dataset/002d24f0-0056-4176-b55e-171ba7f0e0d5/lidar-composite-dtm-2017-2m>.

Code availability

The code for the Total Absolute Difference (TAD) scaling analysis is available as a Minimum Working Example (MWE) on the Open Science Framework (OSF) at https://osf.io/pqx2n/files/osfstorage?view_only=9091e5ac3d7a4aed91aa9945cccbfffe. During the peer-review process, the full repository is hosted privately to maintain anonymity; however, a complete version will be made publicly available on GitHub upon the final publication of this work.

Author contribution

Conceptualization: W.C.B.C. Investigation: W.C.B.C. and R.B. Visualization: W.C.B.C. Supervision: R.B. Discussion: W.C.B.C., M.H.L., and R.B. Writing—original draft: W.C.B.C. Writing—review and editing: W.C.B.C., M.H.L., and R.B.

The Complexity of Urban Verticality: Supplementary Information

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Supplementary Tables

Supplementary Table S1 The population and economic (gross value added and gross domestic product) statistics of the 15 cities, ordered by population size. The first three columns (population, job count, GVA) were collected from Centre for Cities [1], GDP (per head) data was collected from the Office for National Statistics, United Kingdom [2].

City	Population	Job count	GVA (£)	GDP (£)
London	9,350 K	4,986 K	343.53 B	395.3 K ³
Birmingham	2,420 K	1,018 K	44.08 B	28.0 K
Manchester	2,365 K	1,076 K	46.89 B	44.8 K ⁴
Newcastle	829 K	383 K	16.3 B	25.7 K ²
Sheffield	810 K	341 K	14.38 B	24.6 K
Bristol	691 K	386 K	19.1 B	34.4 K
Liverpool	612 K	290 K	12.82 B	30.0 K
Leicester	480 K	226 K	9.36 B	25.9 K
Middlesbrough	465 K	180 K	7.8 B	20.0 K ²
Southampton	362 K	170 K	8.76 B	33.7 K
Preston	356 K	185 K	7.36 B	36.0 K ²
Hull	256 K	115 K	4.7 B	26.3 K
Wolverhampton ¹	249 K	158 K	4.62 B	20.8 K
York	198 K	106 K	4.6 B	33.6 K
Cambridge	124 K	90 K	4.54 B	33.0 K ²

¹The first four columns data (population, job count, GVA, GVA per worker) for Wolverhampton was collected from Office for National Statistics, United Kingdom [3].

²These cities are part of a larger Nomenclature of Territorial Units for Statistics-3 (NUTS3) area, which are Tyneside (Newcastle), South Teesside (Middlesbrough), Mid Lancashire (Preston), & Cambridgeshire CC (Cambridge).

³The GDP for London refers to the NUTS3 Camden and City of London area.

⁴The GDP for Manchester refers to the NUTS3 Manchester area only, which excludes the outer area of the Greater Manchester.

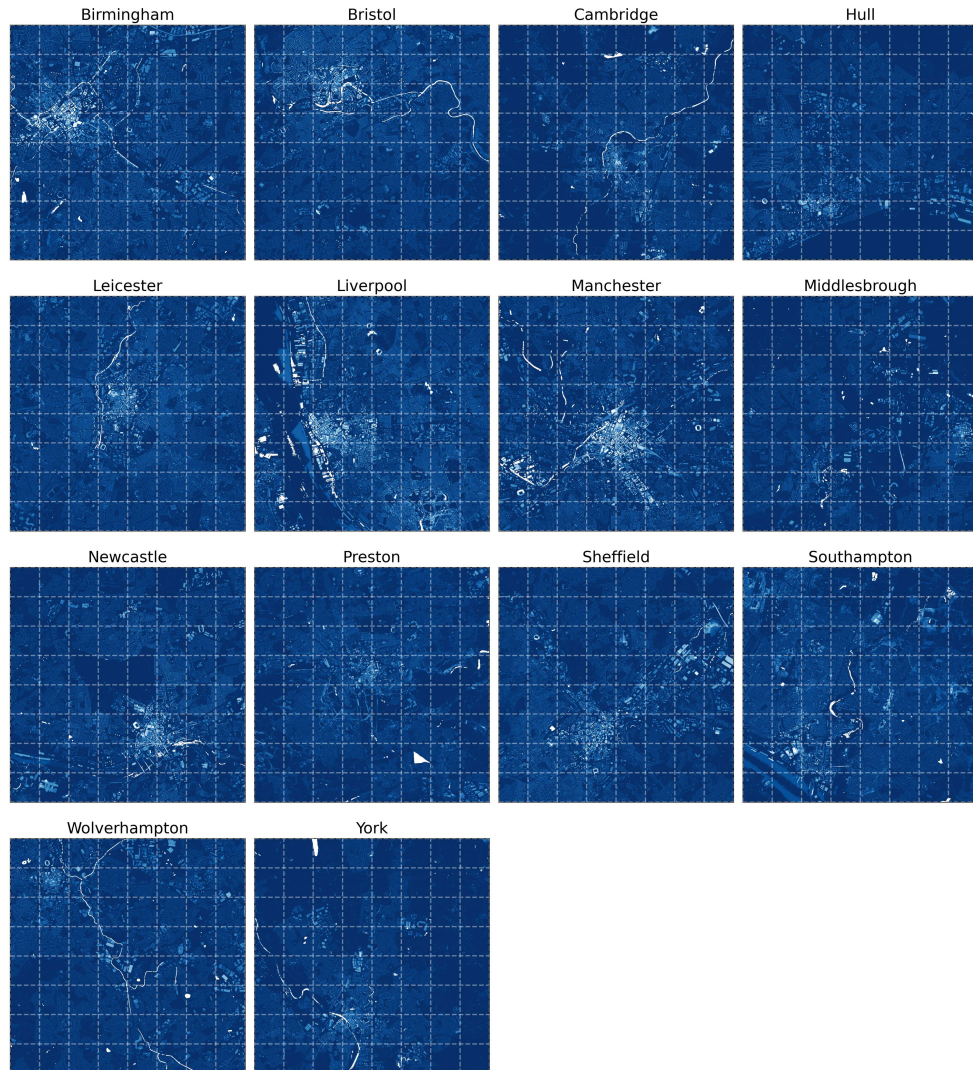
Supplementary Table S2 Welch ANOVA of tail section slopes for three zones in London. Due to unequal variances (Homoscedasticity $p = 0.020$), a robust ANOVA was performed, revealing significant differences across zones ($p \leq 0.001$, $\eta^2 = 0.231$).

Source	ddof1	ddof2	F	p-unc	η^2
city	2	124.382	27.154	0.000	0.231

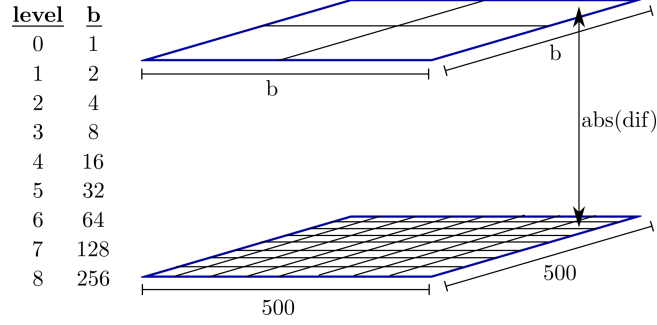
Supplementary Table S3 Games–Howell post-hoc pairwise comparisons of tail section slopes between three London zones. The Games–Howell test was selected to account for unequal variances and sample sizes. Mean differences ($\text{diff} = \text{mean}(A) - \text{mean}(B)$) and Hedges’ g effect sizes are reported, indicating significant differences ($p \leq 0.001$) across all zone pairings.

A	B	$\text{mean}(A)$	$\text{mean}(B)$	diff	se	T	df	$p\text{-val}$	hedges
West	Central	-0.612	-0.640	0.028	0.008	3.569	124.682	0.001	0.627
West	East	-0.612	-0.575	-0.036	0.009	-4.005	121.836	0.000	-0.704
Central	East	-0.640	-0.575	-0.064	0.009	-7.358	116.699	0.000	-1.293

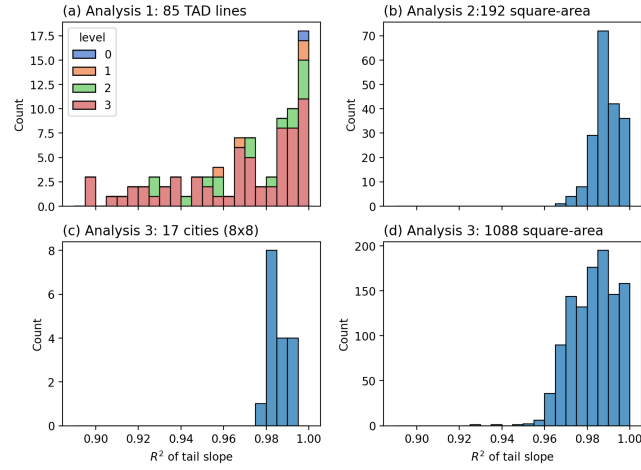
Supplementary Figures



Supplementary Fig. S1 The normalized DSM (nDSM) of the 14 city-centred urban-zone windows in England. The cells in every window correspond to a 1 km² area. The color scale ranges from dark (low height) to bright (high height).



Supplementary Fig. S2 Illustration of the calculation for TAD. The upper layer represents the boxes with average heights from a scale level (e.g., level 1 with 4 boxes). The lower layer is the actual data matrix (D) that is composed by 500×500 cells of fixed size ($2 \times 2m^2$).



Supplementary Fig. S3 Distribution of R^2 values for the tail slope regression across different analysis scales. (a) R^2 results for Analysis 1 (85 TAD lines) categorized by level, showing strong linear alignment even at fine scales. (b) R^2 results for the 192 analysis windows in Analysis 2. (c) R^2 values for the 17 urban-zone windows in Analysis 3. (d) Distribution of all 1,088 R^2 results for the individual analysis windows in Analysis 3. The overwhelming majority of R^2 values exceed 0.95, with a peak distribution near 0.99, confirming that the tail scaling behavior follows a robust power-law relationship.

Supplementary References

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