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Tunneling characteristics of a contact between a superlattice and non-Fermi liquid using the AdS/CFT correspondence

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In this paper, we investigate the peculiar features of the tunneling contact between a superlattice and a non-Fermi quantum liquid. The imaginary part of the Green functions are responsible for the non-Fermi liquid density of states. The Green functions have been derived within the framework of the anti-de Sitter/conformal field theory correspondence.

Keywords: Ads/CFT correspondence; tunnel contact; non-Fermi liquids.

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1. Introduction

Ideas originating from the string theory, in particular, the idea of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence recently contributed to a significant progress in the study of quantum critical phenomena.[1,2] The progress made in recent years is particularly due to the use of techniques that were developed in the string theory.[3,4] Since the 1970s, the investigators of critical phenomena have clearly understood the role of universality, scaling, and hence the importance of ideas from the conformal field theory (CFT). Close connections between critical phenomena and the CFT lead to the clearer understanding and coherent development of both fields.[5,6]

Great progress in calculating the Green's functions for different systems — with various types of transformation of the corresponding fields when changing the coordinates (scalar, spinor, etc.) — was made thanks to the AdS/CFT correspondence between the conformal field theory and the theory of superstrings.[8,9] This fact allows for an experimental verification of some of the associated consequences using methods borrowed from solid state physics. Indeed, the progress of solid-state physics — as a theory in which the key players are inter-particle interactions — occurred largely in parallel with the development of the physics of strong interaction. However, because of tenuous computational difficulties, the development of the theory of strong interaction has divided into several branches, with the leading positions occupied by the string theory and the conformal field theory. While the string theory circumvents several issues by conveniently increasing the dimension of the space, the conformal field theory allows to operate with well-known Green's functions, albeit significantly simplifying the calculations by considering them in an area where the behavior of the Green's functions bears an essentially simpler character. The AdS/CFT correspondence merely indicates a deep connection between these branches of the theory of strong interaction and allows us to set not only ideological, but purely technical relations, linking values computed in just one of the theories.

Recently, the research in this area has primarily focused on the study of conductivity.[10] At the same time there are still a number of characteristics that can be subjected to experimental tests. Among others, the current–voltage characteristics are of special interest when considering a contact between some well-studied material and the test substance. In this case, as will be shown below, the current–voltage characteristics is determined by the imaginary part of Green's functions. Additional incentive to this problem is provided by the fact that devices, based on the tunneling effect, have became an essential part of the element basis of modern electronics, and are of great practical use. Note that mainly the use of these devices is based on the asymmetric current–voltage characteristics and corresponds to the case of weak enough external fields. In the case of large external alternating fields, theoretical analysis becomes difficult since the system is far from equilibrium.

All the above-mentioned facts give us a reason to hope for a success in the description of tunneling effects in quantum critical phenomena. In this paper, we present such an attempt based on the description of tunneling characteristics using Green's functions together with the relations for the quantum critical region taken from the AdS/CFT theory.

2. Basic Equations

To study the tunneling effects, let us define the Hamiltonian of our model as

$$\mathcal{H} = \sum_p \mathcal{E}_p^A a_p^\dagger a_p + \sum_q \mathcal{E}_q^B b_q^\dagger b_q + \sum_{pq} T_{pq}(a_p^\dagger b_q + b_q^\dagger a_p)\,, \tag{1}$$

where $a_p^\dagger$ and a_p are the creation and annihilation operators for metal electrons with momenta p, $\mathcal{E}_p^A$ is the electronic spectrum in the metal, T_{pq} stands for the matrix element of the tunneling operator between the states p and q; $b_q^\dagger$ and b_q are the creation and annihilation operators for the electrons in a substance brought in contact with the metal, and $\mathcal{E}_q^B$ is the electron spectrum of the latter substance. The tunneling current can easily be expressed as

$$J = ie\sum_{pq}(a_p^\dagger b_q - b_q^\dagger a_p)\,, \tag{2}$$

and let us consider the following gauge transformation:[11,13]

$$a_p \to S^{-1}a_p S, \quad S = \exp\left(ieVt\sum_p a_p^\dagger a_p\right),$$

where V is the voltage applied to a test substance and e is the elementary charge. We can formally reduce the problem of calculating the current–voltage characteristics to the calculation of a response for the operator

$$J_t = ie\sum_{pq}(a_p^\dagger b_q e^{ieVt} - b_q^\dagger a_p e^{-ieVt})\,,$$

to an external distortion

$$\mathcal{H}_t = \sum_{pq} T_{pq}(a_p^\dagger b_q e^{ieVt} + b_q^\dagger a_p e^{-ieVt})\,.$$

In the framework of the Green–Kubo theory, the result reads:[11]

$$J = 4\pi e|T|^2\sum_{pq}\int_{-\infty}^{\infty}\mathrm{Im}[G^A(p,E+eV)G^B(q,E)]\{n_f(E)-n_f(E+eV)\}dE\,,$$

where G^A is the Green's function for a superlattice, G^B is the one for a non-Fermi liquid, and $n_f(E)$ is the equilibrium Fermi distribution. Using the Sokhotskyi–Plemelj formula

$$\mathrm{Im}\frac{1}{x+i0} = -\pi\delta(x)\,,$$

we readily obtain the following expression for the tunneling current:

$$J = 4\pi e|T|^2\int_{-\infty}^{\infty}\nu_A(E+eV)\nu_B(E)\{n_f(E)-n_f(E+eV)\}dE\,, \tag{3}$$

where

$$\nu_A(E) = \sum_p \delta(E-E_p^A), \quad \nu_B(E) = \sum_q \delta(E-E_q^B)\,.$$

Here $\delta(x)$ stands for the Dirac delta-function, $\nu_{A,B}(E)$ is the so-called tunnel density of states. Here and thereafter, we are working within the framework of a "rough" contact, i.e. we assume $T_{pq} = T$.[11,12]

The dispersion law for a superlattice reads

$$E_p^A = \epsilon_0 - \Delta\cos(p)\,, \tag{4}$$

where ϵ_0 is the energy of quantum well electrons, Δ is the tunneling integral determined by the overlap of the electron wave functions of neighboring wells, the momentum p being assumed to be directed along the z-axis. As a result of this dispersion law, we come to the following expression:

$$J = 4\pi e|T|^2 \int_{-\infty}^{\infty} \left\{1 - \left(\frac{\epsilon_0 - E - eV}{\Delta}\right)^2\right\}^{-1/2} \mathrm{Im}\, G^B(q,\epsilon)\{n_f(E) - n_f(E+eV)\}dE\,. \tag{5}$$

Thus, the problem is now solely reduced to the computation of the integral in Eq. (5).

3. Excitation Spectrum and the AdS/CFT Correspondence

In the case of the systems such as "strange metals", the Green's functions near the quantum critical point can be found using the holographic approach. High-temperature superconductors in the quantum critical region, as well as a number of other similar materials and non-Fermi liquids, appear to be dual to the models of superstring theory with gravity.[14,15] Here, duality has to be understood in the sense of the theory of gravity with matter (fermions, scalar fields) in spacetime, with a certain dimension being equivalent (in terms of mean values) to some field theory on the boundary of spacetime. This provides us with the opportunity to readily derive the expression for the mean values of the dual variables (understood in the usual sense of the conformal field theory) on the boundary, and, consequently, the Green's functions[1,16–20] by calculating the wave functions of the perturbation fields in the bulk. In Ref. 21, it has been proposed to calculate the Green's functions for "strange metals" in the case of zero temperature, considering the spacetime metric tensor of the form

$$ds^2 = L^2\left(\frac{dt^2 - d\mathbf{x}^2}{r^2} - \frac{dr^2}{r^2}\right),$$

where $d\mathbf{x}^2$ denotes the rest part of metrics, r being the variable over which the transition to the border is performed. The limiting cases $r \to \infty$ and $r \to 0$ correspond to the "horizon" and the "border", respectively. L is a constant with the dimension of a length. Let us proceed with the traditional form of action for the fermion fields

$$S[\psi] = \int d^{d+1}\mathbf{x}\sqrt{-g}\{i\bar{\psi}\Gamma^\mu(\partial - \Omega_\mu - ieA)\psi + m\bar{\psi}\psi\}\,,$$

where d is the dimension, g is the metric tensor, Ω_μ is the spin connection for fermions in a curved spacetime, Γ are the Dirac gamma-matrices, m stands for the mass of fermions, and A is the gauge field.

In the nonzero temperature case, the metric can be defined through

$$ds^2 = \frac{L^2}{r^2}\left(f(r)dt^2 - \frac{dr^2}{f(r)} - d\mathbf{x}^2\right),$$

where $f(r) = 1 - (r/r_0)^2$, r_0 being the parameter determining the temperature in the dual theory through $T = d/4\pi r_0$. The generalization with a chemical potential can be done by representing $f(r)$ as

$$f(r) = 1 - \left(1 + \frac{r^2 + \mu^2}{\gamma^2}\right)\left(\frac{r}{r_+}\right)^d + \frac{r^2 + \mu^2}{\gamma^2}\left(\frac{r}{r_+}\right)^{2(d-1)}, \tag{6}$$

where μ stands for the chemical potential and r_+ is the horizon radius. The parameter γ defines the temperature through

$$T = \frac{1}{4\pi r_+}\left(d - \frac{(d-2)r_+^2\mu^2}{\gamma^2}\right). \tag{7}$$

Following the concepts laid out in Refs. 8 and 17, we find that the imaginary part of the Green's function is given by

$$\operatorname{Im} G^B(\omega) = -h\omega^{2\nu}\sin\theta\left\{h^2\omega^{4\nu}\sin^2\theta + \left(\frac{\omega}{v_f} + h\omega^{2\nu}\cos\theta - k + k_f\right)^2\right\}^{-1}, \tag{8}$$

where v_f is the Fermi velocity, k_f is the Fermi momentum, ν, θ, and h are the positive constants which can in principle be calculated analytically,[21] k is the absolute value of momentum, and ω stands for the frequency.

4. Numerical Analysis and Discussion

The current–voltage characteristics calculated according to Eqs. (5) and (8) is shown in Fig. 1 for different values of the constant ν. As can be seen from the results, the value of the parameter ν strongly influences the behavior of the current–voltage characteristics (CVC). The parameters for the superlattices were taken for typical gallium arsenide structures: ϵ_0 is of the order of tenths of an electron volt, Δ is typically of the order of tenth of electron volts. The critical exponent ν can in general be calculated,[21] but we have varied it in this study to demonstrate the general tendency. Note the presence of the decreasing area of CVC, which indicates an existence of a region of negative differential conductivity. Moreover, with increasing ν another area of positive differential conductance appears. This fact, in turn, creates an opportunity for the experimental verification of the AdS/CFT theory from the data on the resistance of tunnel junctions between superlattices and non-Fermi quantum liquids.

Let us further investigate the influence of parameters of the superlattice on the CVC contact. Typical CVCs for different values of the tunnel integral Δ are shown in Fig. 2.

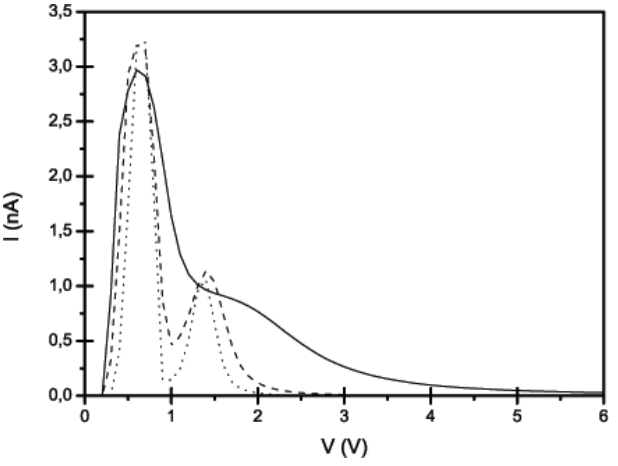

Fig. 1. Current–voltage characteristics of the tunneling contact for $\epsilon_0 = 0.3$ and $\Delta = 0.1$ and the different values of the parameter ν: solid — $\nu = 1$; dashed — $\nu = 2$; dotted $\nu = 3$.

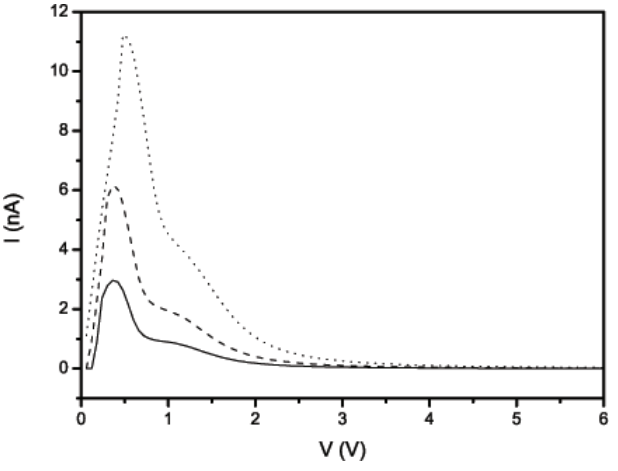

Fig. 2. Current–voltage characteristics of the tunneling contact for $\epsilon_0 = 0.3$ and $\nu = 1$ and the different values of the parameter Δ: solid — $\Delta = 0.1$; dashed — $\Delta = 0.2$; dotted $\Delta = 0.5$.

It appears that the current–voltage dependence remains qualitatively unchanged. Behavior of the curve is determined by the tunneling integral — the higher the value, the greater the current.

CVC dependence on the electron energy ϵ_0 is presented in Fig. 3. From the observed dependence, it can be concluded that with increasing ϵ_0, the maximum value of the tunneling current remains virtually unchanged. The only changes apply to the voltage at which the current begins to flow.

In conclusion, the basic principles of the AdS/CFT theory permit experimental verification using methods of solid state physics by measuring the tunneling

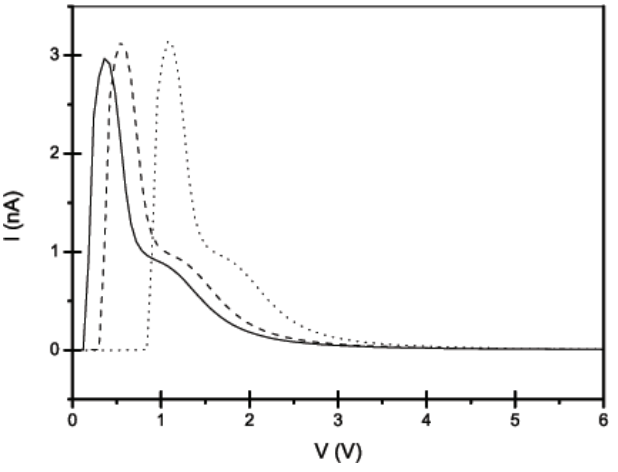

Fig. 3. Current–voltage characteristics of the tunneling contact for $\Delta = 0.1$ and $\nu = 1$ and the different values of the parameter ϵ_0: solid — $\epsilon_0 = 0.3$; dashed — $\epsilon_0 = 0.6$; dotted $\epsilon = 1.5$.

characteristics of the contact of a superlattice and a non-Fermi liquid. The parameter ν influences the current–voltage characteristics most strongly. It is important to note that our analysis is only valid for the area in which the quasiparticles are "well defined" (i.e. the imaginary part decreases faster than the real one). It can also be applied to the area where the quasiparticles are poorly defined, but only close to the Fermi level. The calculations of tunneling characteristics away of the Fermi level represent a separate problem altogether.

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