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Stabilization of ultrashort pulses by external pumping in an array of carbon nanotubes subject to piezoelectric effects

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ABSTRACT

We study the combined effects of electromagnetic pumping and piezoelectric damping on the propagation of ultrashort pulses in carbon nanotubes. Based on Maxwell's equations, an effective equation is obtained for the vector potential of the electromagnetic field, which takes into account both the dissipation of the pulse field associated with piezoelectric effects due to the oscillations of the heavy nuclei of the medium and the pumping from an external electromagnetic wave. Our analysis shows that, when the dissipative piezoelectric effects are properly compensated through external pumping, a stable propagation of the ultrashort pulses is achievable. Specifically, we demonstrate the stability of the steady-state form of the electromagnetic pulse at long time scales with variations in various system parameters, including the absorption coefficient of heavy ions as well as the initial pulse field distribution. In addition, the stability of the pulse with respect to angular perturbations—breaking the axisymmetry of the pulse distribution—is substantiated.

I. INTRODUCTION

The study of phenomena arising from the interaction of electromagnetic radiations with matter is particularly important in modern opto- and nanoelectronics owing to a large number of possible practical applications.[1–5] Recent successes in modern laser technology—e.g., in generating powerful electromagnetic radiations such as ultrashort laser pulses with durations corresponding to several half-periods of the field oscillations[6,7]—have stimulated systematic studies of the propagation of electromagnetic waves, including extremely short pulses, in various media (e.g., see Refs. 8–12). Particular interest in the study of the propagation of ultrashort pulses comes from four key factors: (1) the high directivity of their radiation, (2) the stability of their shape, (3) their resilience to a range of disturbances from certain parameters, and (4) the practical aspects associated with the fact that the peak intensity of their field is sufficient for useful nonlinear properties to manifest without affecting the physical integrity of the waveguide material.[13]

Physical effects arising from the propagation of ultrashort pulses in nonlinear media can be used as the basis for the creation of new energy transfer systems, optical information processing, and other promising compact devices required in modern optoelectronic devices based on various micro- and nanostructures. In this context, graphene-based materials have attracted significant attention given the wide range of applications they offer in terms of both fundamental research and commercial applications (see, for example, Refs. 14–16). In particular, carbon nanotubes—quasi-one-dimensional carbon macromolecules[17–20]—have a high potential for the development of optoelectronic devices based on the propagation of nonlinear electromagnetic waves, such as ultrafast lasers, photodetectors, solar energy converters, transparent conductive surfaces, and displays. Nowadays, carbon nanotubes (CNTs) continue to be the subject of close attention by scientists and engineers due to a number of unique physical properties used in various applications.[21–27] The interest in CNTs is primarily due to the peculiarity of their electronic structure. In particular, the nonparabolicity of the dispersion law of conduction electrons—i.e., the dependence of the energy on the quasimomentum—determines the nonlinearity of the response of nanotubes to applied electromagnetic radiations of moderate strengths, starting with intensities of 10^3–10^4 V/cm. This circumstance allows us to observe a number of unique electromagnetic phenomena in media with nanotubes, including the propagation of solitonlike ultrashort pulses (e.g., see Ref. 28). So far, the propagation of ultrashort electromagnetic pulses in arrays of semiconductor carbon nanotubes have been systematically studied taking into account the influence of various physical factors. In particular, effective equations describing the evolution of the electromagnetic field during the passage through a CNT array were obtained and enabled the study of the effects of impurities and the Coulomb interaction of electrons on the pulse dynamics, the collision of light bullets, and the effect of external fields on the shape of an electromagnetic pulse.[29–34]

It is worth noting that the above-mentioned studies of the propagation of ultrashort pulses in CNT arrays were carried out based on one fairly conservative assumption: namely, that the dissipation of the energy of the ultrashort pulse is considered to be negligible. Such an assumption requires the introduction of strict restrictions on the parameters of the system under consideration. In particular, conditions were imposed on the ratio of the pulse duration to the relaxation time in the electron subsystem, thereby providing the time interval during which the simulation results could be considered fair. The fulfillment of this criterion assumed that the relaxation time significantly exceeds the pulse duration but is still shorter than the system observation time.[31] As a consequence, it appears timely to consider the generalization of the mentioned model by introducing various dissipative factors present in real systems (e.g., see Ref. 35). Indeed, the search for conditions associated with the stabilization of the propagation of ultrashort pulses in dissipative systems at large times becomes a critical task.

The possibility of a stable propagation of ultrashort pulses in semiconductor waveguides may be an important prerequisite for the development of new and more advanced methods of transmitting and processing data, whose carriers may be electromagnetic solitons. Such a technical process can be achieved using dissipative solitary waves, which are more stable than conservative solitons.[36,37,40] In Ref. 41, the stable propagation of three-dimensional (3D) ultrashort pulses in an array of carbon nanotubes with two-level impurities was demonstrated. In this work, it was shown that an inverse population of levels makes it possible to counterbalance the attenuation of the pulse field under the action of dissipative factors. In Ref. 42, external field pumping was proposed as a way to alleviate the unavoidable dissipative effects that lead to the damping of the pulse field. It was shown that, in principle, the propagation of three-dimensional extremely short pulses can be sustained in CNTs by pumping energy into the pulse through an external electromagnetic field. At the same time, it is also critical ensuring the stability of ultrashort pulses subjected to disturbances—a key feature of solitary waves with the properties of dissipative solitons.[36–39] In this paper, we study the interaction of an ultrashort pulse with the medium of an array of semiconductor carbon nanotubes under damping conditions due to piezoelectric effects associated with the oscillations of the heavy nuclei of the medium. To compensate the dissipation and stabilize the pulse, the system is irradiated by an external electromagnetic pump wave. We show that an ultrashort pulse that forms in the system under consideration retains its stability, even in the presence of disturbances that break the axisymmetry of pulse.

II. FORMULATION OF THE PROBLEM AND GOVERNING EQUATIONS

We consider the propagation of three-dimensional ultrashort electromagnetic pulses in an array of zigzag carbon nanotubes. For definiteness, we assume that the electromagnetic pulse propagates along the axis of the nanotubes (z-direction), and its electric vector field is collinear to the Oy-axis (Fig. 1).

The potential vector has the form $\mathbf{A} = \{0, A(x, y, z, t), 0\}$, the electric current density $\mathbf{j} = \{0, j(x, y, z, t), 0\}$, and the polarization of the medium $\mathbf{P} = \{0, P(x, y, z, t), 0\}$. For the component of the electric field, directed along the axis of the CNTs (taking into account the Lorentz gauge $\mathbf{E} = -\frac{1}{c}\partial\mathbf{A}/\partial t$), we write the three-dimensional wave equation as

$$\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} - \Gamma\frac{\partial A}{\partial t} = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} + 4\pi j(A) - \beta\frac{\partial P}{\partial t}, \quad (1)$$

where the parameter $\Gamma > 0$ describes the pumping of the electric field[43] and, accordingly, its amplification and c is the speed of light in vacuum. To take into account the properties of the medium, Eq. (1) contains a term involving the rate of change of the polarization of the medium P (rightmost term), which is directed along the CNT axis (Fig. 1). The pumping here is introduced phenomenologically and, in general, depends only on the spatial coordinates. Also, via the parameter $\beta = 4\pi/c$, we phenomenologically take into account the reverse effect of the medium excited by an electric field pulse on the electric field itself.

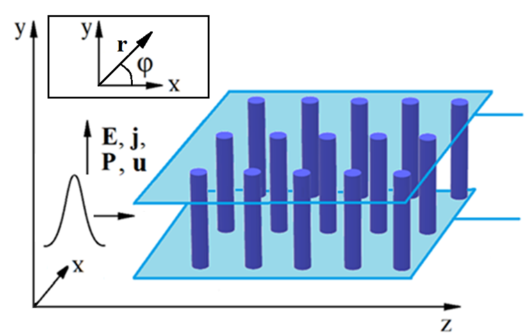

FIG. 1. Schematic diagram of the setup and the associated coordinate system.

This problem is better reformulated using a cylindrical coordinates system,

$$\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}-\Gamma\frac{\partial A}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right)+\frac{\partial^2 A}{\partial z^2}+\frac{1}{r^2}\frac{\partial^2 A}{\partial \phi^2}+4\pi j(A)-\beta\frac{\partial P}{\partial t}, \tag{2}$$

with cylindrical coordinates (r, ϕ, z), so that $r^2 = x^2 + y^2$. The standard expression for the current density reads[31]

$$j = 2e\sum_{s=1}^{m}\int_{\mathrm{BZ}} v_s(p)f(p, s)dp, \tag{3}$$

where e is the elementary charge, p is the projection of the quasi-momentum of the conduction electron along the axis of the nanotube (Oy-axis), $v_s(p) = \partial\epsilon_s(p)/\partial p$ is the electron velocity, $f(p, s)$ is the Fermi distribution, and $\epsilon_s(p)$ is the dispersion law, which describes the properties of electrons of CNTs of zigzag type $(0, m)$ and has the form[17,22]

$$\epsilon_s(p) = \pm\gamma_0\left\{1+4\cos(ap)\cos\left(\frac{\pi s}{m}\right)+4\cos^2\left(\frac{\pi s}{m}\right)\right\}^{1/2}, \tag{4}$$

where $s = 1, 2, \ldots m$, $\gamma_0 \approx 2.7$ eV, $a = 3b/2\hbar$, and $b = 0.142$ nm is the distance between adjacent carbon atoms. In Eq. (3), the integration is carried out over the first Brillouin zone (BZ).

Generally, the nonuniform character of the ultrashort pulse propagating in an array of CNTs induces nonuniformity in the medium, thereby resulting in charges accumulated in some areas. However, earlier calculations[31] showed that this effect of charge accumulation for femtosecond pulses can be neglected. As a consequence, we can safely assume that the axisymmetry of the field distribution is preserved. Based on this, we assume that all derivatives with respect to the angle ϕ are zero. As a result, we obtain the following effective equation for the vector potential:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right)+\frac{\partial^2 A}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 A}{\partial t^2}+\frac{4en_0}{c}\sum_{q=1}^{\infty}b_q\sin\left(qa\frac{e}{c}(A+\eta)\right)f(t)$$
$$+\Gamma\frac{\partial A}{\partial t}-\beta\frac{\partial P}{\partial t}=0, \tag{5}$$

where n_0 is the electron concentration and the parameter η determines the displacement vector of the medium,

$$f(t) = \begin{cases} 0, & t < t_0(z), \\ \exp(-t/t_{\mathrm{rel}}), & t \geq t_0(z), \end{cases} \tag{6}$$

where $t_0(z) \approx (z - z_0)/v$ is the time at which the intensity of the pulse at its leading edge, measured at the point with the z coordinate, is e times less than the peak intensity of the pulse; z_0 is the initial coordinate of the "center of mass" of the pulse at the initial time $t = 0$; $v \approx c/\sqrt{k_0}$ is the approximate pulse velocity; k_0 is the average relative dielectric constant of the medium (array of nanotubes); and t_{rel} is the relaxation time of the electron subsystems. The coefficients b_q in Eq. (5) are given by

$$b_q = \sum_s a_{sq}\int_{\mathrm{BZ}}\cos(pq)\frac{\exp\{-\epsilon_s(p)/k_BT\}}{1+\exp\{-\epsilon_s(p)/k_BT\}}dp, \tag{7}$$

where k_B is the Boltzmann constant, T is the temperature, and a_{sq} is the coefficients in the expansion of the electron dispersion law (4) as a Fourier series,

$$\epsilon_s(p) = \frac{1}{2\pi}\sum_{s=1}^{m}\sum_{q=1}^{\infty}a_{sq}\cos(pq), \tag{8}$$

$$a_{sq} = \int_{\mathrm{BZ}}\cos(pq)\varepsilon_s(p)dp. \tag{9}$$

Due to a decrease in the coefficients b_q with an increase in q [see Eq. (7)], we can restrict ourselves to the first 15 nonvanishing terms in Eq. (9)[28] and obtain the generalized sine-Gordon equation,[44] which is widely used in applications but not integrated by the inverse scattering method.

The value of η in Eq. (7) is related to the nonzero component of the displacement vector of the medium $\mathbf{u} = \{0, u(z, t), 0\}$ as[45]

$$\eta = -cd\int_{-\infty}^{t}\frac{\partial u(z, t')}{\partial z}dt', \tag{10}$$

where d is the piezoelectric strain coefficient. Here, we consider the simplest model, in which the polarization of the medium is directly proportional to the applied field owing to the piezoelectric effect,

$$P = d\frac{\partial u}{\partial z}. \tag{11}$$

In this case, Eq. (10) must be supplemented with an equation for the nonzero component of the displacement vector u,[45,46]

$$\frac{\partial^2 u}{\partial t}+\gamma\frac{\partial u}{\partial t}+\omega_0^2 u = \chi\frac{\partial A}{\partial t}. \tag{12}$$

Here, γ is the absorption coefficient of the nuclei of the medium (heavy ions), ω_0 is the resonant frequency of vibrations of heavy ions, and χ is the susceptibility coefficient. It is worth highlighting that absorption by such nuclei can significantly narrow the range of transparency and reduce the width of the transmission spectrum in a medium of carbon nanotubes.[35]

Within the framework of the model described, a number of remarks must be made. First, we consider only one component of the displacement vector, which can easily be generalized. Second, we do not take into account that the medium may have nonlinear acoustic properties and, as a result, the polarization vector may be noncollinear to the electric field vector.

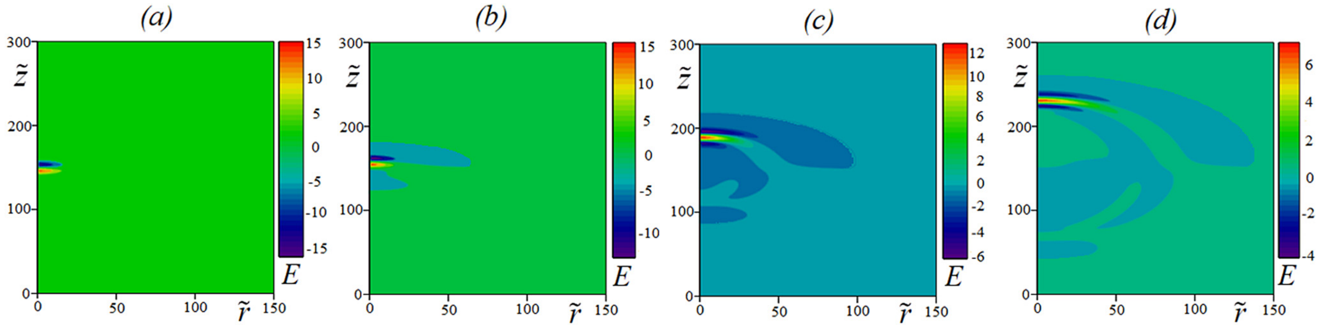

FIG. 2. Evolution of a pulse for the case of a single oscillation of the electric field E for $Q_\Gamma = 1$: (a) $\tilde{t} = 0$; (b) $\tilde{t} = 1.0$; (c) $\tilde{t} = 5.0$; and (d) $\tilde{t} = 9.0$. The nondimensional unit of E corresponds to 10^7 V/m.

III. NUMERICAL RESULTS

The basic governing equations (5) and (12) are solved numerically using an explicit difference scheme of the "cross" type.[47] For convenience, we introduce the following dimensionless variables and parameters (Planck's constant is taken as unity):

$$\tilde{A} = \frac{eaA}{c}, \quad \tilde{r} = \frac{r}{a}, \quad \tilde{z} = \frac{z}{a}, \quad \tilde{t} = \frac{ct}{a}, \quad \tilde{\eta} = \frac{ea\eta}{c}, \quad \tilde{u} = \frac{eau}{cd}. \tag{13}$$

The initial condition for the vector potential can be written in the following dimensionless form:

$$\begin{gathered}\tilde{A}(\tilde{r},\tilde{z},0) = Q\tilde{r}^{n-1}\exp\left\{-\frac{(\tilde{z}-\tilde{z}_0)^2}{\tilde{l}_z^2}\right\}\exp\left(-\frac{\tilde{r}^2}{\tilde{l}_r^2}\right),\\ \frac{d}{d\tilde{t}}\tilde{A}(\tilde{r},\tilde{z},0) = 2v\frac{(\tilde{z}-\tilde{z}_0)}{\tilde{l}_z^2}Q\tilde{r}^{n-1}\exp\left\{-\frac{(\tilde{z}-\tilde{z}_0)^2}{\tilde{l}_z^2}\right\}\exp\left(-\frac{\tilde{r}^2}{\tilde{l}_r^2}\right),\\ \tilde{u}(\tilde{r},\tilde{z},0) = 0, \quad \frac{d}{d\tilde{t}}\tilde{u}(\tilde{r},\tilde{z},0) = 0,\end{gathered} \tag{14}$$

where Q is the amplitude of the electromagnetic pulse at the entrance to the CNT-based medium; v is the initial pulse velocity when entering the medium; $\tilde{l}_z$ and $\tilde{l}_r$ determine the pulse width along the z- and r-directions, respectively; $\tilde{z}_0$ is the initial coordinate of the center of the pulse along the z-axis; $n = 1$ for one oscillation of the electric field; and $n = 2$ is the pulse profile for two oscillations of the electric field. The initial condition corresponding to the Gaussian pulse profile with two oscillations of the electric field is represented by the following expression:

$$\begin{gathered}\tilde{A}(\tilde{r},\tilde{z},0) = Q(\tilde{z}-\tilde{z}_0)\exp\left\{-\frac{(\tilde{z}-\tilde{z}_0)^2}{\tilde{l}_z^2}\right\}\exp\left(-\frac{\tilde{r}^2}{\tilde{l}_r^2}\right),\\ \frac{d}{d\tilde{t}}\tilde{A}(\tilde{r},\tilde{z},0) = Q\exp\left\{-\frac{(\tilde{z}-\tilde{z}_0)^2}{\tilde{l}_z^2}\right\}\exp\left(-\frac{\tilde{r}^2}{\tilde{l}_r^2}\right)\left\{-\frac{v}{\tilde{l}_z^2} + 2v\frac{(\tilde{z}-\tilde{z}_0)}{\tilde{l}_r^2}\right\}.\end{gathered} \tag{15}$$

The evolution of the electromagnetic field as it propagates over the sample in the case of a single oscillation of the electric field is shown in Fig. 2.

The attenuation of the pulse can clearly be observed, as well as the appearance of a "tail" in its wake. The stabilization of the pulse can be achieved by tuning the parameter Γ, which is responsible for the pumping of the electric field, in the super-Gaussian form,

$$\Gamma(\tilde{r}) = Q_\Gamma \exp\left(-\frac{\tilde{r}^6}{\tilde{l}_\Gamma^6}\right). \tag{16}$$

Here, $\tilde{l}_\Gamma$ determines the width of the amplifying medium in the direction perpendicular to the direction of propagation of the electric pulse. Note that the choice of a super-Gaussian form in Eq. (16) for the parameter Γ originates from our need to compensate the diffraction spreading of the pulse. Hence, outside the

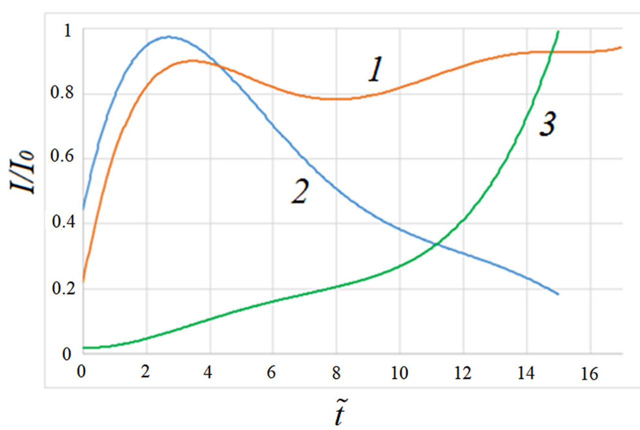

FIG. 3. Evolution with time of the pulse intensity for the case of a single oscillation of the electric field. (Curve 1) $Q_\Gamma = 1.5$ and a balance between attenuation and amplification is achieved; (Curve 2) $Q_\Gamma = 1.0$ and dissipation prevails; and (Curve 3) $Q_\Gamma = 2.0$ and amplification through pumping prevails. I_0 is the peak intensity for each of the three cases.

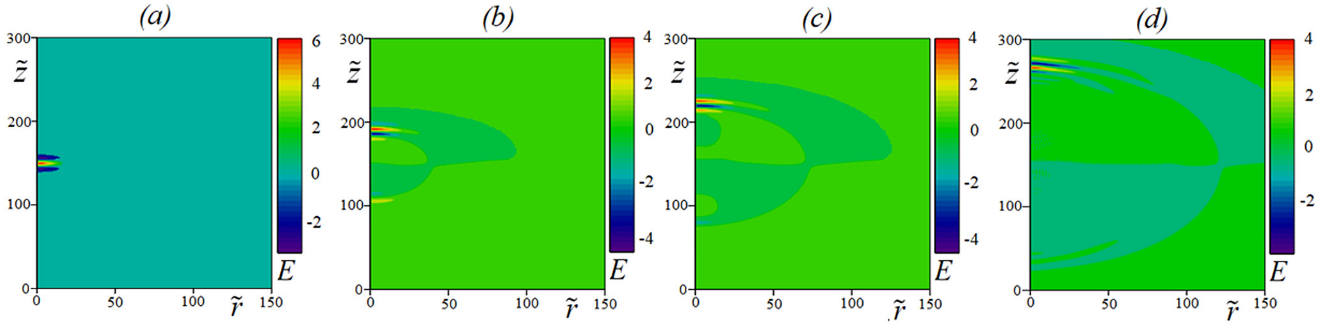

FIG. 4. Evolution of a pulse for the case of two oscillations of the electric field E for $Q_\Gamma = 1$: (a) $\tilde{t} = 0$; (b) $\tilde{t} = 5.0$; (c) $\tilde{t} = 9.0$; and (d) $\tilde{t} = 15.0$. The nondimensional unit of E corresponds to 10^7 V/m.

region in which the amplification takes place, the pulse will experience the usual attenuation due to the inherent dissipative effects within the medium. The prefactor Q_Γ is a phenomenological parameter introduced to characterize the properties of the amplification process within the CNT-based medium.

A close observation of curve 1 in Fig. 3—corresponding to an intermediate value of $Q_\Gamma = 1.5$, the intensity of the pulse reaches a plateau over time. Thus, it can be concluded that the effects of the linear dissipative factors can be overcome through field pumping, with ultimately a stabilization of the pulse. For a pulse consisting of two field oscillations, a result similar to that in Fig. 2 is obtained and shown in Fig. 4. The numerical analysis shows that there is a range of values for the amplification and dissipation parameters, leading to a conditional stabilization of the pulse. In other words, changes in the amplitude of the pulse are quite small over a long time interval.

To better appreciate the evolution of the pulse shape as it propagates, we present in Fig. 5 the dynamics of the pulse width with time. Specifically, the pulse width (denoted L) is taken as the distance between its transverse boundaries, corresponding to a decrease in the amplitude of 50%. By studying a number of cases corresponding to different types of initial pulse profile, we conclude that the most stable pulses are those that initially possess one field oscillation. The temporal evolution of the maximum absolute value of the pulse amplitude is shown in Fig. 6. Over time, the pulses consisting of one-field oscillation exhibit a stabilization of their amplitude. This result clearly favors selecting one field oscillation for the initial profile of the electromagnetic pulse.

Moreover, we investigated the behavior of a pulse in the form of a ring with the axis of coinciding with the direction of propagation of the pulse (z-axis). The maximum in distribution of the field of such a pulse is situated at $r \neq 0$ (see Fig. 7).

The evolution of the pulse shape with time is shown in Fig. 8, in the particular case of a pumping pulse with a ring profile. Such solutions are quite close to the vortex solutions reported in Refs. 8 and 48 which, in general, may be unstable. However, we note that the pulse of a ring shape in our case is quasistable. The outcome of various analyses (see Figs. 2, 4, and 8) confirms that, although diffraction spreading of the pulse occurs in the direction transverse to the direction of propagation, the pulse generally

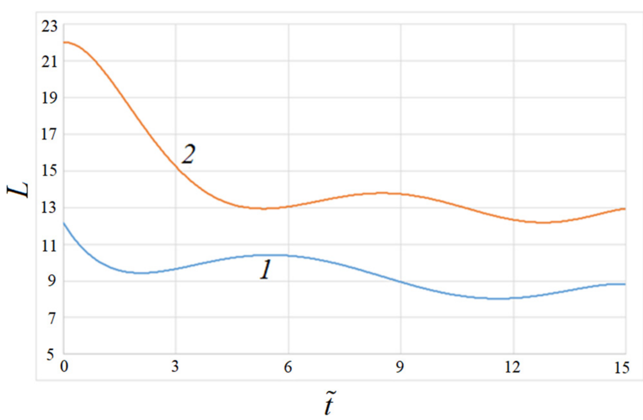

FIG. 5. Temporal evolution of the pulse width L for different initial conditions. (Curve 1) one field oscillation and (Curve 2) two field oscillations.

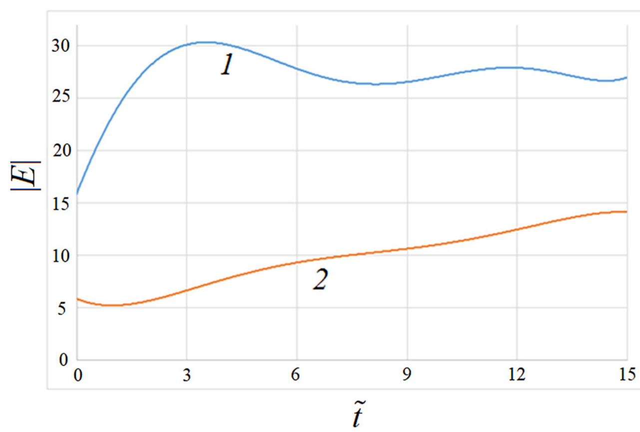

FIG. 6. Temporal evolution of the maximum absolute value of the pulse amplitude for different initial conditions. (Curve 1) one field oscillation and (Curve 2) two field oscillations. The nondimensional unit of E corresponds to 10^7 V/m.

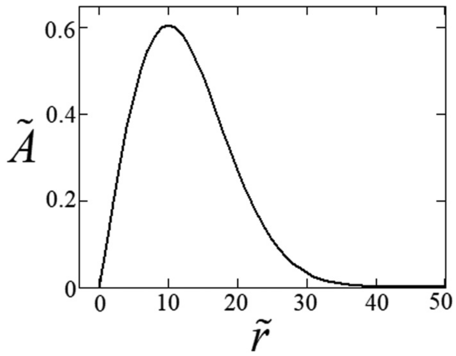

FIG. 7. Profile of the ring-pulse at $t = 0$.

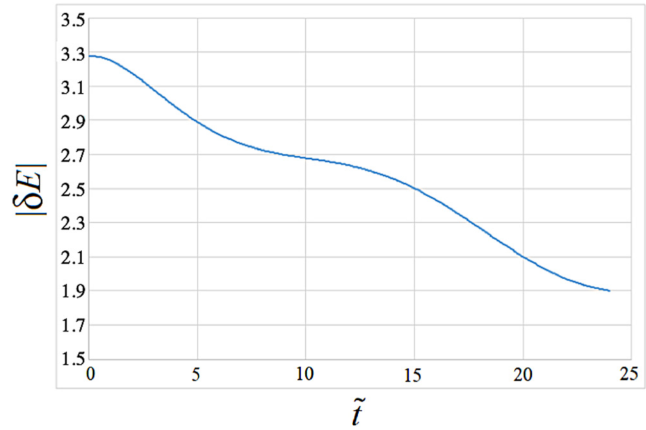

FIG. 9. Temporal variations of the maximum of $|\delta E|$ (with $n = 3$). The nondimensional unit of E corresponds to 10^7 V/m.

retains its shape. It should also be noted that a partial distortion of the pulse front occurs due to diffraction.

An important issue is the study of the stability of the obtained solutions with respect to small-amplitude perturbations of the pulse field E which depends on the angle ϕ. The stability analysis can be carried by linearizing Eq. (2) considering $A = A_0 + \delta A$, where δA are small-amplitude perturbations of the potential vector,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \delta A}{\partial r}\right)+\frac{\partial^2 \delta A}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 \delta A}{\partial t^2} + \frac{4en_0}{c}\sum_{q=1}^{\infty} b_q \cos\left\{\frac{aeq}{c}(A_0+\eta)\right\}\delta A\frac{aeq}{c}\exp\left(-\frac{t}{t_{\mathrm{rel}}}\right) + \Gamma\frac{\partial \delta A}{\partial t}-\beta\frac{\partial P}{\partial t}=0. \tag{17}$$

Note that the last term in (17)—related to the induced polarization—is calculated based on the solution $A_0(z, r, t)$ of Eq. (5). By virtue of the linearity of Eq. (17), one can consider perturbations made up of modes of the form

$$\delta A = \delta A(z, r, t)\exp(in\phi). \tag{18}$$

We can then calculate the corresponding corrections to the electric field by means of the Lorentz gauge condition,

$$\delta E = -\frac{1}{c}\frac{\partial \delta A}{\partial t}. \tag{19}$$

Equation (17) is solved numerically with the following initial conditions:

$$\delta\tilde{A}(\tilde{r},\tilde{z},0)=\delta Q\tilde{r}^{n-1}\exp\left\{-\frac{(\tilde{z}-\tilde{z}_0)^2}{\tilde{l}_z^2}\right\}\exp\left(-\frac{(\tilde{r}-\tilde{r}_0)^2}{\tilde{l}_r^2}\right),$$

$$\frac{d}{d\tilde{t}}\delta\tilde{A}(\tilde{r},\tilde{z},0)=2\nu\frac{(\tilde{z}-\tilde{z}_0)}{\tilde{l}_z^2}\delta Q\tilde{r}^{n-1}\exp\left\{-\frac{(\tilde{z}-\tilde{z}_0)^2}{\tilde{l}_z^2}\right\}\exp\left(-\frac{(\tilde{r}-\tilde{r}_0)^2}{\tilde{l}_r^2}\right),$$

$$\tilde{u}(\tilde{r},\tilde{z},0)=0,\ \frac{d}{d\tilde{t}}\tilde{u}(\tilde{r},\tilde{z},0)=0. \tag{20}$$

Figures 8 and 9 show the corresponding results for the electric field. These figures show the temporal variations of the maximum absolute value of δE (in the entire computational domain) depending on the mode n.

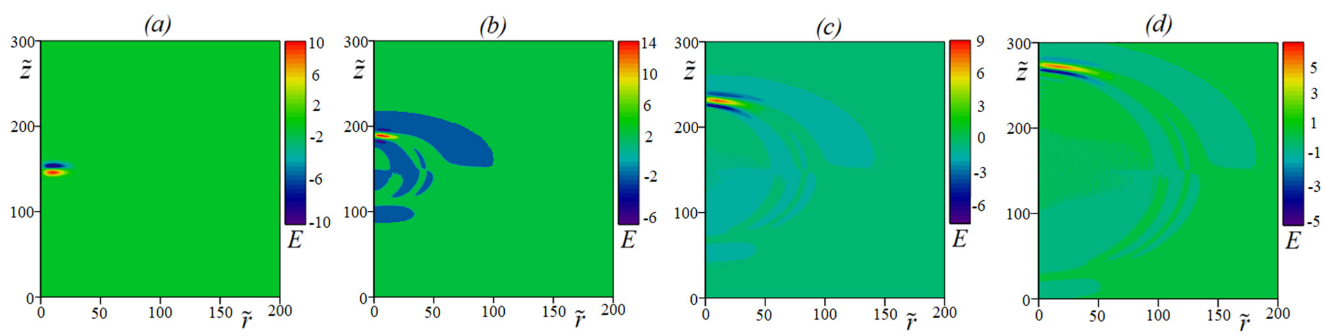

FIG. 8. Evolution of the pulse for the initial profile of the pulse in the form of a ring with $Q_\Gamma = 1$: (a) $\tilde{t} = 0$; (b) $\tilde{t} = 5.0$; (c) $\tilde{t} = 10.0$; and (d) $\tilde{t} = 15.0$. The nondimensional unit of E corresponds to 10^7 V/m.

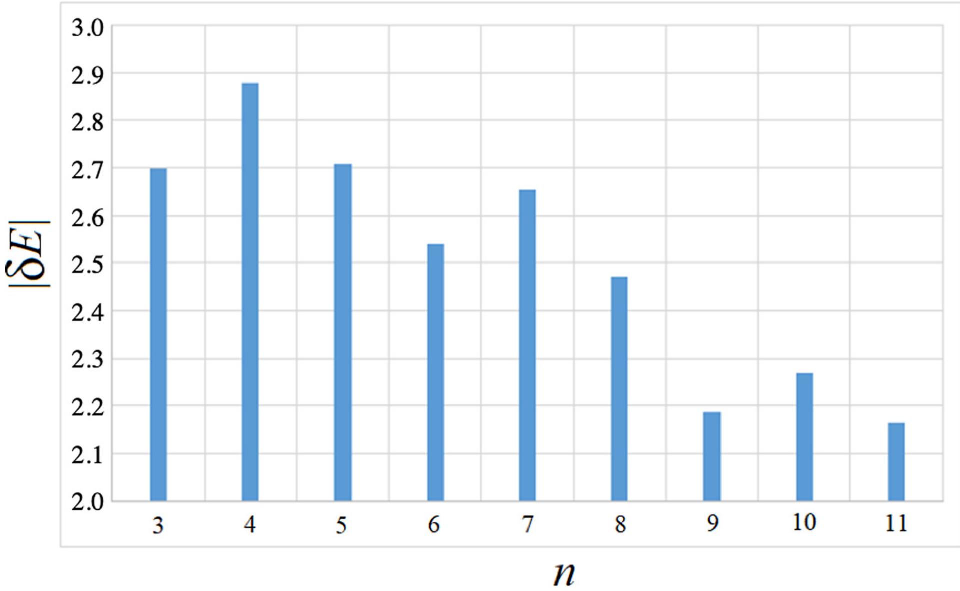

FIG. 10. Dependence of the maximum of $|\delta E|$ on n (at $\tilde{t} = 10$). The nondimensional unit of E corresponds to 10^7 V/m.

Figure 8 shows that the amplitude of the perturbations monotonically decays with time, and this behavior appears to be stable and robust at large times. According to Fig. 9, higher-order modes (i.e., higher values of n) experience a more rapid decay. Figures 8 and 9 allow us to conclude that the solutions obtained are stable with respect to perturbations in angle ϕ. Hence, we can conclude that having steady and stable propagation of 3D ultrashort and localized pulses in arrays of CNTs when providing appropriate external pumping is a possibility to overcome attenuation induced by the absorption of heavy ions (Fig. 10).

IV. CONCLUSIONS

This study considers external field pumping as a way to counteract the dissipation of the pulse field associated with piezoelectric effects due to the oscillations of the heavy nuclei in the CNT-based medium. The key results obtained may be summarized as follows:

(i) Due to the change in the amplitude of the pumping pulse, it is possible to control the shape of an ultrashort optical pulse and stabilize it. The amplitude of the pumping field is the key factor affecting the propagation of electromagnetic pulses in the CNT arrays.

(ii) We uncover the possibility of a stable propagation of an extremely short electromagnetic pulse when pumping appropriately counterbalances attenuation. Indeed, dispersion spreading of pulses during their propagation can be compensated by means of inhomogeneous pumping along the sample diameter, which taps into the nonlinearity of the CNT-based medium.

(iii) We demonstrate that ultrashort pulses propagating in the array of CNTs are stable with respect to modal angular perturbations and that they maintain the axisymmetry of the field distribution in the space of the system under consideration.

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