AIP Journal of Applied Physics

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Citation: Journal of Applied Physics **115**, 203109 (2014); doi: 10.1063/1.4879900
View online: http://dx.doi.org/10.1063/1.4879900
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/115/20?ver=pdfcov
Published by the AIP Publishing

JOURNAL OF APPLIED PHYSICS **115**, 203109 (2014)

Interaction of a two-dimensional electromagnetic breather with an electron inhomogeneity in an array of carbon nanotubes

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(Received 9 April 2014; accepted 14 May 2014; published online 29 May 2014)

Propagation of ultrashort laser pulses through various nano-objects has recently became an attractive topic for both theoretical and experimental studies due to its promising perspectives in a variety of problems of modern nanoelectronics. Here, we study the propagation of extremely short two-dimensional bipolar electromagnetic pulses in a heterogeneous array of semiconductor carbon nanotubes. Heterogeneity is defined as a region of enhanced electron density. The electromagnetic field in an array of nanotubes is described by Maxwell's equations, reduced to a multidimensional wave equation. Our numerical analysis shows the possibility of stable propagation of an electromagnetic pulse in a heterogeneous array of nanotubes. Furthermore, we establish that, depending on its speed of propagation, the pulse can pass through the area of increased electron concentration or be reflected therefrom. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4879900]

I. INTRODUCTION

One of the most promising objects for modern nanoelectronics is the ensemble of carbon nanotubes (CNTs),[1] which represent quasi-1D macromolecules of carbon. The nonlinearity of the electron dispersion in nanotubes leads to a wide range of properties, which manifest in fields of moderate intensity $\sim 10^3 - 10^5$ V/cm (see, e.g., Refs. 2 and 3 and references therein). Recent successes in laser physics in the generation of powerful electromagnetic radiations with given parameters,[4] have provided the impetus for comprehensive studies of electronic and optical properties of CNTs in the presence of an electromagnetic field. Special interest in phenomena related to the propagation of ultra-short electromagnetic pulses through an array of CNTs[5–10] has recently been mounting. In particular, the possibility for propagation of solitary electromagnetic waves in an array of CNTs has been demonstrated,[5,6] as well as the dynamics of a periodic train of electromagnetic pulses, and the induced current domains have been investigated.[7,8]

Generally, the theoretical and experimental studies of electromagnetic solitary waves have a rather long history. In the last two decades, optical solitons in Bose–Einstein condensates have attracted a growing body of interest (see Refs. 11 and 12 and references therein for a review on this topic). More generally, the propagation of stable/quasistable optical solitons requires specific types of medium nonlinearity.[13–15] The latter can be provided, e.g., by a proper choice of nonlinear lattices (topic comprehensively reviewed in Ref. 16). In this context, the CNT arrays provide one of the unique and practically reliable systems for studying various aspects of nonlinear electromagnetic waves propagation in media.

Earlier studies of the propagation of electromagnetic pulses were mainly devoted to the analysis of 1D cases. Later on, it was realized that there are a lot of unresolved issues remaining in 2D and 3D cases, some of which are quite peculiar. Indeed, the possibility of propagation of cylindrically symmetric electromagnetic waves in an array of nanotubes has been demonstrated.[17] Furthermore, the possibility of propagation of 2D traveling solitary electromagnetic waves (a.k.a. *light bullets*) has been reported in Ref. 18. Subsequently, their interaction with inhomogeneities in the arrays of nanotubes has been investigated.[19–21] The possibility of propagation of 2D bipolar electromagnetic pulses in semiconductor arrays of CNTs was revealed in Ref. 22. The general aspects of stability of the laser beams propagating in an array of CNTs were analyzed in Ref. 23.

It should be noted that the theoretical analysis in the abovementioned studies has been performed under the assumption of homogeneity of the pulse field along the axis of the CNTs. However, the heterogeneity of this field can cause the emergence of interesting and unexpected physical effects of potentially practical importance. In particular, Ref. 24 is concerned with the 2D model of the propagation of ultrashort electromagnetic pulses in an array of CNTs with the heterogeneity of the field along their axis. Furthermore, recently the authors carried out a comprehensive study of the latter problem in the fully 3D case, which resulted in the demonstration of the possibility of 3D bipolar electromagnetic breathers propagation through an array of CNTs with account for the field inhomogeneity.[25] As a result, it was found that in that specific case, an electromagnetic pulse induces a significant redistribution of the electron density in the sample, both in 2D and 3D systems.

Apart from the field inhomogeneity leading to the electrons redistribution, there are other natural heterogeneities in the experimental samples. A case of special importance is when heterogeneities are caused by regions of increased conduction electron concentration, induced by the presence of impurities. In this regard, it seems appropriate to study the effects of heterogeneity of the electron concentration on the

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characteristics of propagation of extremely short bipolar electromagnetic pulses through an array of semiconducting carbon nanotubes. The relevance of this issue is critical for modern optoelectronics applications.

The paper is constructed as follows. The effective equation describing the evolution of the electric field during the propagation of two-dimensional extremely short bipolar electromagnetic pulses through an inhomogeneous array of semiconductor CNTs is derived in Sec. II. In Sec. III, we describe the details of our numerical simulation scheme and discuss the obtained results. The main outcomes of our study are summarized in Sec. IV.

II. EFFECTIVE WAVE EQUATION

Let us consider the propagation of a solitary electromagnetic wave (laser pulse) through a volumetric array of monolayer semiconductor carbon nanotubes of the zigzag type, $(m,0)$, where the number m (not a multiple of three) determines the radius of the nanotube through $R = \sqrt{3}bm/2\pi$, with b the distance between nearest-neighbor carbon atoms.[3] This particular choice of CNTs type is dictated by the presence of stable solitary solutions even for the generic sine-Gordon case in 2D.[26] We assume that the nanotubes are placed into a homogeneous insulator in a way that the axes of the nanotubes are parallel to the common Ox-axis, and the distances between neighboring nanotubes are large compared to their diameter, which allows us to neglect the interaction between CNTs.[22,23]

Given the above framework, the dispersion relation for the conduction electrons of CNTs takes the form

$$\Delta(p_x, s) = \gamma_0 \left\{ 1 + 4\cos\left(p_x \frac{d_x}{\hbar}\right)\cos\left(\pi \frac{s}{m}\right) + 4\cos^2\left(\pi\frac{s}{m}\right)\right\}^{1/2}, \tag{1}$$

where the quasimomentum is represented as $\mathbf{p} = \{p_x, s\}$, with $s = 1, 2, \ldots, m$ is the number characterizing the momentum quantization along the perimeter of the nanotube, γ_0 is the overlap integral, and $d_x = 3b/2$.[2,3]

Now, let us consider the propagation of a laser beam in an array of CNTs in the direction perpendicular to the their axes, namely, along the Oz-axis with our choice of spatial coordinates. Further, we assume that the electric field of the laser beam, $\mathbf{E} = \{E(y,z,t), 0, 0\}$, is oriented along the Ox-axis. The characteristic pulse duration τ_p is supposed to satisfy the condition $\tau_p \ll \tau_{rel}$, where τ_{rel} is the characteristic relaxation time. The latter condition allows us to use the collisionless approximation to describe the evolution of the pulse field.[5] The electromagnetic field in an array of nanotubes can be described by Maxwell's equations,[28] which in our case lead to the following wave equation for the vector potential:

$$\frac{\varepsilon}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \frac{\partial^2 \mathbf{A}}{\partial y^2} - \frac{\partial^2 \mathbf{A}}{\partial z^2} - \frac{4\pi}{c}\mathbf{j} = 0, \tag{2}$$

where $A(y, z, t)$ and $j(y, z, t)$ are the projections of the vector potential $\mathbf{A} = (A, 0, 0)$ and the current density $\mathbf{j} = (j, 0, 0)$ onto the Ox-axis, ε is the permittivity of the medium, and c is the speed of light in vacuum. The electric field of the laser beam is determined by the gauge condition $\mathbf{E} = -c^{-1}\partial\mathbf{A}/\partial t$.[29]

In this particular study, we assume that the electric field along the Ox-axis is homogeneous. The inhomogeneity of the field can lead to a charge redistribution, so that it is necessary to include additional induced fields into consideration.[25] The latter problem is beyond the aim of the present study. Let us derive the conduction current density in an array following the approach developed in Refs. 22, 23, and 31. Following the scheme used in, e.g., Ref. 10, and representing the electron energy spectrum (1) as a Fourier series, we write the expression for the projection of the current density on the Ox-axis in the collisionless approximation, namely,

$$j = -en\frac{d_x}{\hbar}\gamma_0 \sum_{s=1}^{m}\sum_{\alpha=1}^{\infty} G_{\alpha,s}\sin\left\{\alpha\frac{ed_x}{c\hbar}A\right\}, \tag{3}$$

where e is the electron charge, n is the concentration of conduction electrons in the array of nanotubes, and the coefficients $G_{\alpha,s}$ are explicitly given by

$$G_{\alpha,s} = -\alpha\frac{\delta_{\alpha,s}}{\gamma_0}\frac{\int_{-\pi}^{\pi}\cos(\alpha\xi)\exp\left\{-\sum_{\alpha=1}^{\infty}\theta_{\alpha,s}\cos(\alpha\xi)\right\}d\xi}{\int_{-\pi}^{\pi}\exp\left\{-\sum_{\alpha=1}^{\infty}\theta_{\alpha,s}\cos(\alpha\xi)\right\}d\xi}, \tag{4}$$

with $\theta_{\alpha,s} = \delta_{\alpha,s}(k_B T)^{-1}$, and $\delta_{\alpha,s}$ are the coefficients in the expansion of the spectrum (1) in a Fourier series

$$\delta_{\alpha,s} = \frac{d_x}{\pi\hbar}\int_{-\pi\hbar/d_x}^{\pi\hbar/d_x}\Delta(p_x,s)\cos\left(\alpha\frac{d_x}{\hbar}p_x\right)dp_x. \tag{5}$$

Details of the derivation of Eq. (3) can be found in Ref. 10. In our work, the dispersion law is expanded as a Fourier series. At this point, we have to note that the current density in Eq. (3) explicitly depends on the vector potential $\mathbf{A}$. Therefore, it might be assumed that the change of the vector potential by adding a scalar constant (which is classically known not to lead to any physical consequence) causes a change in the current density. However, in reality, this does not happen, because while deriving Eq. (3), it was assumed that the vector potential $\mathbf{A}$ initially (at $t = 0$) is zero valued, which fixes its choice.

It is worth noting that the field is not known *a priori* in the general case. That means that our present formalism is not universal. However, we are not seeking general solutions to this problem. Instead, we are considering the specific problem of the propagation of a very short nonlinear solitary pulse (breather). Therefore, in our framework, the preset characteristics of the initial pulse are assumed to be precisely known.

Let the area with a high concentration of electrons be situated along the Ox-axis of the array of nanotubes. In the simplest case, it can be modeled using a step function.[30] Thus, with account of the expression for the current conduction (3), Eq. (2) gives us the wave equation describing the field in an array of nanotubes containing region with an elevated concentration of electrons

$$\frac{\partial^2\Phi}{\partial\tau^2}-\left(\frac{\partial^2\Phi}{\partial\nu^2}+\frac{\partial^2\Phi}{\partial\zeta^2}\right)+\eta(\nu,\zeta)\sum_{s=1}^{m}\sum_{\alpha=1}^{\infty}G_{\alpha,s}\sin(\alpha\Phi)=0. \tag{6}$$

Here, $\Phi = Aed_x/c\hbar$ is the dimensionless projection of the vector potential on the Ox-axis; $\tau = \omega_0 t/\sqrt{\varepsilon}$ is the dimensionless time; $\nu = y\omega_0/c$ and $\zeta = z\omega_0/c$ are the dimensionless coordinates; $\eta = n/n_0$, where n_0 is the equilibrium electron concentration in the absence of external fields; and ω_0 is the characteristic frequency, defined by the formula

$$\omega_0 = 2\frac{ed_x}{\hbar}\sqrt{\pi\gamma_0 n_0}. \tag{7}$$

The step function $\eta(\nu,\zeta)$ used in Eq. (6) has the form

$$\eta(\nu,\zeta)=\begin{cases}1, & \text{homogeneous area}\\ n/n_0, & \text{inhomogeneous area}\end{cases}. \tag{8}$$

This choice of the inhomogeneity distribution—fully characterized by Eq. (8)—originates from our wish to understand the basic peculiarities of the pulse behavior in the presence of some inhomogeneities. The latter are always present because of impurities, structural defects, etc. Obviously, the step function in Eq. (8) does not represent a realistic configuration. However, it conveniently offers us the possibility to understand the underlying physics. Using our framework, more realistic charge distributions could be considered in future studies.

Note that we assume our system to be homogeneous along the Ox-axis. While deriving Eq. (6), we have used the collisionless approximation, which is valid for times smaller than the characteristic relaxation time $\tau \approx 10^{-12}$ s.[2] This assumption is fully justified by the fact that the typical size of the pulse is of the order of micrometer, while the diameter of CNTs is of the order of 10 nanometers. Even if we consider multilayered CNTs, the overlap constant would not exceed 0.1 eV, while within a layer it is about 3 eV. In the array, the nanotubes are weakly linked, so that the overlap constant does not exceed 0.01 eV, thereby allowing us to neglect all effects appearing during the ultra-short pulse passage.

The electric field of the wave in the array of nanotubes, given its form $\mathbf{E} = (E, 0, 0)$, reads

$$E = -\frac{1}{c}\frac{\partial A}{\partial t} = E_0\frac{\partial\Phi}{\partial\tau}, \tag{9}$$

in which E_0 is given by

$$E_0 = -\frac{\hbar\omega_0}{ed_x\sqrt{\varepsilon}}. \tag{10}$$

One may notice that we are not dealing with the scalar potential φ in our construction. Indeed, initially, the system is electrically neutral. For a given geometry, the current is uniform along the whole length of CNTs, so the charge is not elevated. Hence, the system essentially remains neutral, and the d'Alembertian of the scalar potential is equal to zero with zero boundary conditions. Thus, the governing equation for φ is $\Box\varphi = 0$ ($\Box$ being the d'Alembertian operator) admits only constant solutions, equal to the potential at infinity, chosen to be zero using classical conventions.

As is well known, the practically measurable physical quantities are the intensity, energy, or power of the electromagnetic radiation, which are proportional to the square of the absolute value of the electric field vector.[28] Taking into account the expression (9) and the chosen gauge for a vector potential, the intensity $I = \mathbf{E}^2$ takes the form

$$I = I_0\left(\frac{\partial\Phi}{\partial\tau}\right)^2, \tag{11}$$

where $I_0 = E_0^2$ (see Eq. (10)).

III. NUMERICAL SIMULATION AND DISCUSSION OF RESULTS

Given the quite general framework considered, Eq. (6), in general, does not possess any exact analytical solutions. Hence, we carried out a numerical study of the propagation of an electromagnetic pulse in an array of CNTs. Equation (6) can be considered as a generalization of the 2D sine-Gordon equation. Let us assume that at the instant $t = 0$ the specimen is irradiated by a bipolar electromagnetic pulse, described by its dimensionless vector potential of the form

$$\Phi = 4\arctan\left\{\frac{\sin\chi}{\cosh\mu}\sqrt{\frac{1}{\Omega^2}-1}\right\}\exp\left\{-\left(\frac{\nu-\nu_0}{\lambda}\right)^2\right\}, \tag{12}$$

where we have used the following notations:

$$\chi = \Omega\frac{\tau-(\zeta-\zeta_0)u/v}{\sqrt{1-(u/v)^2}}, \tag{13}$$

$$\mu = \{\tau u/v-(\zeta-\zeta_0)\}\sqrt{\frac{1-\Omega^2}{1-(u/v)^2}}. \tag{14}$$

$\Omega = \omega_B/\omega_0$ is the parameter determined by the natural frequency of the breather ω_B ($\Omega < 1$); u/v is the ratio of the pulse velocity u to the speed of light in the medium $v = c/\sqrt{\varepsilon}$; ν_0 and ζ_0 are the dimensionless coordinates along the Oy- and Oz-axes, respectively, at $\tau = 0$; λ is the dimensionless half-width of the pulse along the Oy-axis.

Note, the first factor in Eq. (12) is a traveling breather, propagating with velocity u.[32] The choice of initial pulse is justified in Refs. 22, 23, and 25. For the numerical solution of Eq. (6), we have implemented explicit finite-difference schemes for hyperbolic equations.[33] Difference scheme steps in both time and space were iteratively decreased twice until the solution became unchanged in the eighth decimal place. To prevent any reflection at the boundaries of the computational domain, an absorbing boundary treatment was implemented similar to what is classically done.[20,22,23,25,26,35] Specifically, this absorbing boundary is achieved by expanding the computational domain beyond the limits of the actual

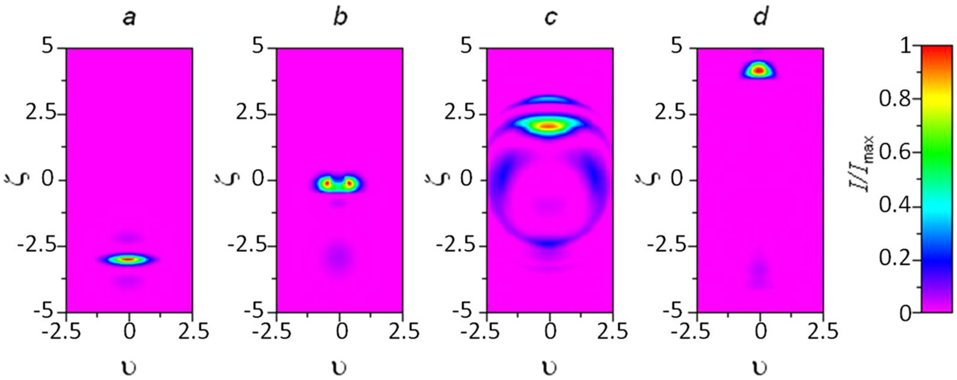

FIG. 1. Field intensity distribution in the array of CNTs at different points of the dimensionless time $\tau = \omega_0 t/\sqrt{\varepsilon}$ during the propagation of laser pulse through the region of elevated electron concentration: (a) $\tau = 0$, (b) $\tau = 4.0$, (c) $\tau = 8.0$, (d) $\tau = 12.0$. The axes are scaled using the dimensionless coordinates $\nu = y\omega_0/c$, and $\zeta = z\omega_0/c$. Values of the ratio I/I_{max} are mapped on a color scale, the maximum values of the field intensity correspond to red, and minimum ones to purple. I_{max} is the maximum value of I at the given instant τ considered.

physical boundaries, and by imposing a variable damping throughout the extended part of the computational domain. The intensity of the damping is increased logistically from zero at the border with the physical part of the computational domain. The number of additional damping layers on the outer part of the computational domain is varied in order to effectively dampen any reflection with the same accuracy as the above convergence criterion. The obtained values for $\Phi(\nu, \zeta, \tau)$ have been further used to calculate the electric field by means of Eq. (9), as well as the intensity distribution of the field through Eq. (11).

Modeling of the laser pulse propagation has been done for an array of carbon nanotubes of the (7, 0)–type, where we have used the following realistic parameters: $\gamma_0 = 2.7\,\text{eV}$, $b = 1.42 \times 10^{-8}\,\text{cm}$, $d_x \approx 2.13 \times 10^{-8}\,\text{cm}$, $n_0 = 2 \times 10^{18}\,\text{cm}^{-3}$,[27] $T = 77\,\text{K}$, and $\varepsilon = 4$. We assume that the array of nanotubes contains heterogeneity, namely a region of elevated electron concentration, $n = 5n_0$. Furthermore, we approximate the latter region by a rectangular shape in the plane $\nu O\zeta$. The characteristic sizes of an inhomogeneity are $\delta\nu = \delta\zeta = 0.5\,(\delta y = \delta z \approx 1.5 \times 10^{-4}\,\text{cm})$, and the center of the region is located at the origin $\nu = \zeta = 0$. For definiteness, we have chosen the following initial parameters: $\Omega = 0.5$ (equivalent to $\omega_B = \Omega\omega_0 \approx 5.05 \times 10^{13}\,\text{s}^{-1}$); $\lambda = 1$ (equivalent to the pulse half-width along the axis Oy equal to $L_y = \lambda c/\omega_0 \approx 3 \times 10^{-4}\,\text{cm}$), $\nu_0 = 0$, and $\zeta_0 = -3$. Numerical simulations show that the result of the interaction of ultrashort laser pulse with an area of elevated electron density depends on the speed of the pulse $u = \beta v$. Figures 1–4 represent the typical results of modeling the pulse propagation in an inhomogeneous array of CNTs.

Figures 1 and 2 illustrate the propagation of an ultrashort laser pulse for the initial velocity $u = \beta c/\sqrt{\varepsilon} = 1.43 \times 10^{10}$ cm/s (which corresponds to $\beta = 0.95$, see Eq. (12)) through the region of elevated electron concentration. Specifically, Fig. 1 represents the distribution of field intensity within the array of nanotubes in a plane $\nu O\zeta$ at different instances of the dimensionless time $\tau = \omega_0 t/\sqrt{\varepsilon}$. The field intensity is presented by a ratio $I(\nu, \zeta, \tau)/I_{max}$, different values of which correspond to a variation of colors (flooded contours) with a colormap from violet to red. The quantity I_{max} stands for the maximum intensity at the given instant τ considered. Red areas correspond to near-maximum intensity, while purple ones reflect near-minimum intensity regions. The distribution is plotted using the dimensionless coordinates, $\nu = y\omega_0/c$ and $\zeta = z\omega_0/c$. With the parameters selected above, the units on the axes correspond to the physical distances $\Delta z = \Delta y \approx 3 \times 10^{-4}$ cm.

Figure 2 represents the intensity distribution $I(\nu_0, \zeta, \tau)/I_0$ of the pulse field in an array of CNTs in an area parallel to the plane $\xi O\zeta\,(\xi = x\omega_0/c)$ and passing through the point $\nu = \nu_0 = 0$, at the same instants of the dimensionless time τ as in Fig. 1. Figure 2 allows us to trace the variation of the laser pulse and the maximum field intensity when the pulse passes through the region of elevated electron density. The key result gathered from Figures 1 and 2 is that the two-dimensional bipolar electromagnetic pulse continues to propagate through an environment after the interaction with the heterogeneity without incurring a significant spreading.

However, the situation changes drastically depending on the initial pulse velocity. Figures 3 and 4 demonstrate the reflection of a laser pulse from the region of the elevated electron density for the initial pulse velocity $u = 1.28 \times 10^{10}$ cm/s (which corresponds to $\beta = 0.85$). Particularly, Fig. 3 shows the intensity distribution on the plane $\nu O\zeta$ at different instants of τ, while Fig. 4 gives us the intensity distribution $I(\nu_0, \zeta, \tau)/I_0$ of the pulse field in an array of CNTs in an area parallel to the plane $\xi O\zeta$, and passing through the center of a pulse. The very interesting outcome seen from Figs. 3 and 4 is that even the reflected pulse remains stable and propagates in the opposite direction without a significant spreading.

Thus, the results of our numerical analysis reveal that depending on the initial velocity, the laser pulse can either pass through a region of increased concentration of electrons or be reflected therefrom. Electromagnetic pulses with low propagation velocities are reflected from a region of enhanced electron density, while the pulses with velocities exceeding a certain threshold value u_c overcome the region

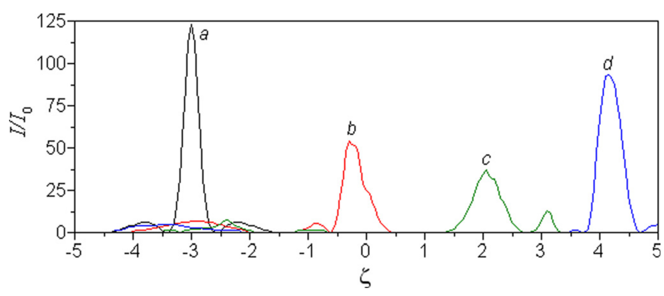

FIG. 2. Intensity distribution $I(\nu_0, \zeta, \tau)/I_0$ of the pulse field in an array of CNTs in an area parallel to the plane $\xi O\zeta\,(\xi = x\omega_0/c,\ \zeta = z\omega_0/c)$ and passing through the point with coordinates $\nu = \nu_0 = 0$, at different instants of the dimensionless time τ: (a) $\tau = 0$, (b) $\tau = 4$, (c) $\tau = 8$, (d) $\tau = 12$.

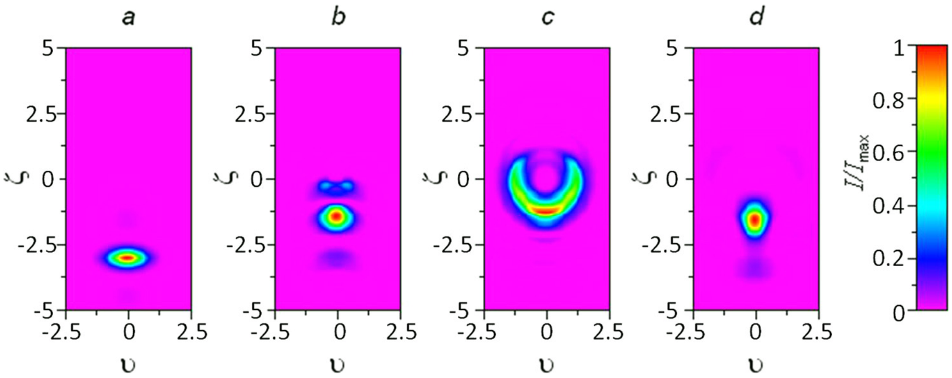

FIG. 3. Field intensity distribution in the array of nanotubes when the laser pulse is reflected from a region of elevated electron density at different instants of the dimensionless time τ: (a) $\tau = 0$, (b) $\tau = 4$, (c) $\tau = 8$, (d) $\tau = 12$. The principle of determining the values of the ratio $I(\nu_0, \zeta, \tau)/I_0$ is the same as in Fig. 1.

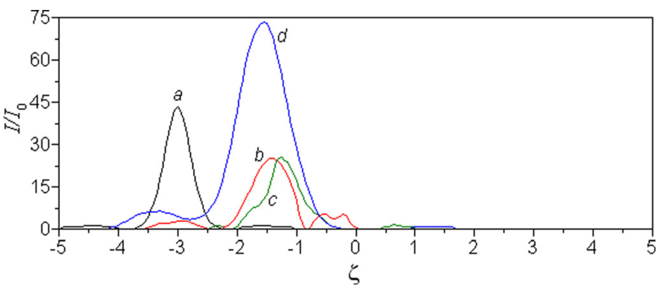

FIG. 4. Intensity distribution $I(\nu_0, \zeta, \tau)/I_0$ of the pulse field in an array of CNTs in a cross-section parallel to the plane $\xi O \zeta$ ($\xi = x\omega_0/c$, $\zeta = z\omega_0/c$) and passing through the point with coordinates $\nu = \nu_0$, when the laser pulse is reflected from a region of elevated electron density at different instants of the dimensionless time τ: (a) $\tau = 0$, (b) $\tau = 4$, (c) $\tau = 8$, (d) $\tau = 12$.

of high electron density. The value of u_c depends on several factors, which include the heterogeneity parameters: The ratio of the concentration of electrons in an anomalous region to the electron density in the homogeneous part of the sample, and the size of the region of the elevated concentration of electrons. The probability of the pulse's passage through the region of high electron density increases with the increase of initial velocity and while reducing the size of the inhomogeneity, ceteris paribus. This result echoes with the results of Ref. 34, revealing the selective nature of the transmission of electromagnetic solitons through the region of high electron density in the quantum semiconductor superlattices. The revealed peculiarities of the interaction of ultrashort bipolar electromagnetic pulses with the region of elevated electron concentration in an array of semiconductor carbon nanotubes can be potentially useful for the development and design of new laser based control devices, optical information processing systems, etc.

IV. CONCLUSIONS

In summary, the key results of our study are the following:

(i) We consider an ultrashort laser pulse, propagating through an array of carbon nanotubes, containing the region of elevated electron concentration. For the first time, we have derived the governing equation describing the evolution of the electric field during the propagation of two-dimensional extremely short bipolar electromagnetic pulses through an inhomogeneous array of semiconductor CNTs (see Eq. (6));

(ii) We have found that the general wave equation (6) can be considered as a generalization of a two-dimensional sine-Gordon equation. Taking the initial pulse being a traveling 2D breather, we have demonstrated its stable propagation through a specimen;

(iii) Depending on the initial velocity, the laser pulse can either pass through the inhomogeneous region, or get reflected therefrom. The characteristic velocity, corresponding to a passing/reflection turnover, generally depends of the size of an inhomogeneity and the density of electrons in the elevated concentration region;

(iv) We have demonstrated that in both cases, whether passing or reflecting, the pulse remains stable and continues to propagate without incurring a significant spreading. The latter observation makes the system under study being potentially useful in various practical applications.

ACKNOWLEDGMENTS

A. V. Zhukov and R. Bouffanais are financially supported by the SUTD-MIT International Design Centre (IDC). M. B. Belonenko acknowledges a support from the Russian Foundation for Fundamental Research.

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