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Roland Bouffanais

Design and Control of Swarm Dynamics

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To my beloved wife Ariane,
and our turbulent, chaotic,
and joyful swarm of boys:
Aurélien, Tristan, and Thibault

Preface

In the past decades, scientists and engineers have been faced with the development and management of an ever-increasing number of distributed systems made of many interconnected components: e.g., the Internet, integrated power grids, transportation networks, cyber-physical systems, fleet of autonomous vehicles, etc. All these systems are complex systems in the sense that they are dynamical systems made of many interacting parts that under certain conditions give rise to spontaneous self-organization. In many cases, their overall collective organization has not been specifically *designed* but instead grew dynamically out of some needs and requirements at the local level: e.g., local area networks, power grid extensions, new road or airport developments, etc.

As humans, we are also becoming increasingly more interconnected: physically thanks to transportation networks, and virtually with the advent of social networking platforms and associated practices. In addition, the unprecedented growth of dense urban environments led to the realization of the crucial need to *control* and regulate dynamic collective behaviors: be they vehicles on the road, airplanes in the vicinity of airports, and even human crowds in high-density areas.

Through self-organization, these complex systems made up of artificial or living units are capable of collectively performing tasks that greatly outperform each individual agent's ability. Thus, the whole becomes greater than the sum of its parts such that the group harnesses swarm intelligence to produce robust and flexible collective actions. On the other hand, under certain conditions, these emergent properties may trigger a disruptive process—often cascading and catastrophic—such as a collapse in collective operation or cooperation, jamming, etc.

This book is about one specific class of complex systems, namely swarming systems in the biological realm, or also multiagent networked systems in the engineering realm. Swarms represent one of nature's most sophisticated achievements in collective operation. As scientists, we are only starting to unlock the secrets of the awe-inspiring dynamics and displays of biological swarms. As engineers, we envision a future filled with specifically designed artificial swarms

performing complex tasks with astonishing effectiveness, robustness, and flexibility.

Devising control laws and design principles for artificial swarms requires a thorough overarching understanding of swarms. This scientific endeavor is truly interdisciplinary as elements from biology, physics, network science, complexity theory, control, information theory, and computation are necessary. This book aims at emphasizing the connections between all these disciplines in order to provide a holistic approach toward the design and control of swarm dynamics.

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Chapter 1
Complexity and Swarming Systems

Complexity science has shown that collective behaviors in animal groups, that is swarms, emerge from repeated local interactions between neighboring individuals. It has also revealed that a set of simple local interaction rules applied to very simple artificial agents gives rise to complex patterns possessing long-range and long-lasting dynamic order. These swarming systems, be them natural or artificial, are all characterized by somehow similar features: (i) lack of central controller or leader overseeing the collective dynamics, instead the latter emerges through self-organization, (ii) local perception of the environment leading to a certain level of global knowledge by means of effective distributed information sharing, and (iii) a high degree of adaptation to rapidly changing circumstances. These features afford swarms unique distributed problem solving capabilities—a.k.a. swarm intelligence—such that collectively they perform tasks far exceeding each individual agent's ability.

As fascinating and impressive biological swarms can be, they have evolved to collectively solve the same problem they have been faced with for millennia: foraging, predator avoidance, nesting, etc. As we will see throughout this book and in Chap. 2 in particular, there is no doubt that they are an invaluable source of inspiration and information about the functioning details of collective behaviors. However, for engineers dealing with a specific real-life task or problem that cannot be solved by classical means, resorting to a swarming approach is often an extremely appealing option, which can turn into some sort of Chinese puzzle given the lack of design rules and principles.

A thorough overarching understanding of swarms is required if one is to seek design principles. This is especially true if the following two prominent features of swarms are sought after: (i) *robustness*—i.e., their ability to continue to operate with missing elements, and (ii) *flexibility*—i.e., their ability to adapt to rapidly changing environments.

This book goes beyond the critical question of the understanding of the mechanistic aspects of swarming, and how these aspects are related to functional considerations for the group as whole. Specifically, this book isaimed at discussing the issue

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of design and control of collective behaviors from a multidisciplinary perspective. We firmly believe that a quest for design rules and principles requires studying all facets of collective behaviors using concepts and elements borrowed from biology, physics, network science, complexity theory, control, information theory, and computation. For instance, we will see in the coming chapters that following repeated physical interactions, swarms are capable of forming a dynamic communication channel taking the form of a complex network allowing social information transmission necessary for distributed problem solving based on collective computation. We also believe that it is equally important to identify and highlight bridges and commonalities between different concepts and representations of these various fields of engineering and science. Even though each chapter of this book is dedicated to a specific approach to studying collective behaviors—from a particular discipline perspective, we have emphasized as much as possible the links and relationships between different parts of the book. This came about naturally given the profound interdisciplinary character of the study of design and control of swarm dynamics.

Given the immense diversity in swarming behaviors, we have opted to focus our attention on discussing the most widely studied, modeled, and probably best understood collective behavior of all: namely a swarm of collectively moving agents seeking consensus in their direction of travel. Despite its relative simplicity, such consensus-reaching collective behaviors display amazingly complex features—some of them hidden from the naked eye—and therefore, ideally epitomize most of the central tenets of swarming. Nevertheless, some specific and more "sophisticated" features of swarms, such as collective learning and cognition, artificial intelligence, or adaptive memory, will only be mentioned without being thoroughly dealt with.

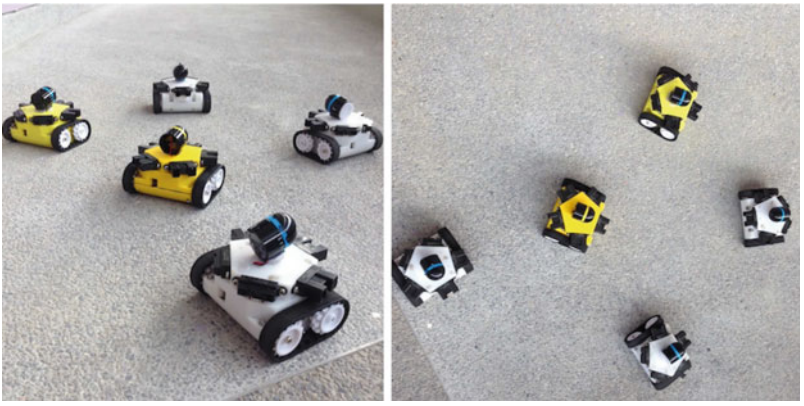

Fig. 1.1 Swarm of land robots "evobots" developed to perform collective surveillance. The bots exchange information about their respective localization, heading, and other sensed data using short-range communications through a mesh network. The distributed control algorithm is loaded onto each bot and collective computation is performed. Various collective behaviors such as heading consensus have been successfully implemented and tested. This swarm of evobots has been jointly developed with Prof. Erik Wilhelm at SUTD in the frame of the STARS project funded by the TL@SUTD

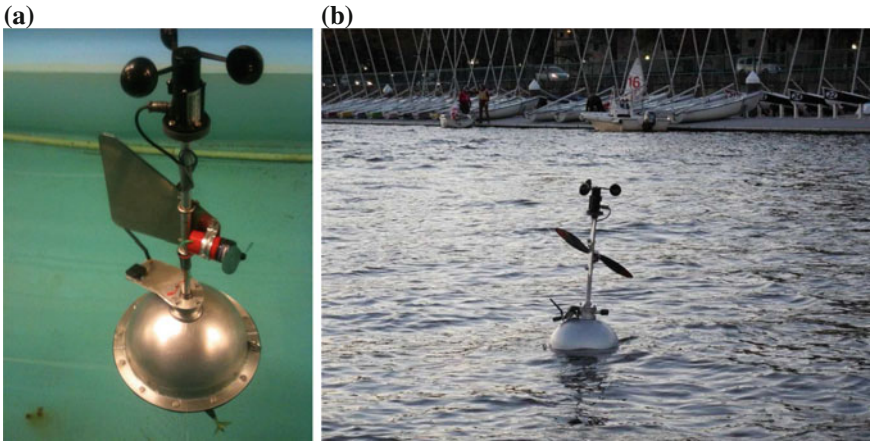

Fig. 1.2 **a** Mobile marine buoy that is GPS-enabled, self-righting, and energy-harvesting. This buoy is loaded with sensors to perform ocean and coastal monitoring. Through long-range (∼2 km) communication, a dynamic network of mobile buoys collectively harvest environmental data. **b** Field testing of one buoy. This swarm of buoys has been jointly developed with Prof. Dick K.P. Yue at MIT in the frame of a project funded by the Center for Environmental Sensing and Modeling (CENSAM) under the Singapore MIT Alliance for Research and Technology (SMART)

Finally, we are convinced that the analysis and discussion presented in this book are timely. Nowadays, virtually anyone can build its own basic swarming system. Indeed, with the advent of low-cost, powerful, and reliable robotic platforms—loaded with combined sensory arrays, capable of wireless communications, and afforded a great deal of computational power—it is now possible to control specifically designed artificial swarms (two such examples are shown in Figs. 1.1 and 1.2).

Chapter 2
A Biologically Inspired Approach to Collective Behaviors

Animal groups provide paradigmatic examples of collective phenomena in which repeated interactions among individuals produce dynamic patterns and responses on a scale larger than individuals themselves. Some of the examples around us include the coordinated movements of fish and birds in a school or a flock, respectively, the chemotactic aggregation of amoebae, the formation of lanes in densely packed human crowds, the generation of vortices in bacterial colonies, the synchronized march of wingless locusts, and the synchronized flashing of fireflies. Many more examples can also be found inside all of us: the firing of neurons in our brains, the clustering of differentiated cells to construct our organs both during embryonic development and wound healing, and the targeted response of neutrophils as part of the initial immune response to a bacterial infection. This nonexhaustive list of collective behaviors of unicellular and multicellular organisms is revealing of the pervasiveness of swarming in the natural world. Thus, there is no better place to start a book dealing with swarms (or flocks, schools, colonies, etc. [1]; the generic term "swarm" will be used interchangeably and loosely throughout this book, although in principle some differences in meanings exist) than by turning to this vast range of awe-inspiring solutions offered by mother nature.

2.1 Collective Animal Behaviors

Like the animals taking part in them, biological collective behaviors come in many different sizes and shapes (see Fig. 2.1). Some like baitballs—comprised small fish swarming in a tightly packed spherical formation about a common center—are simply mesmerizing. A very high level of spatial and social structure is not uncommon in raiding columns of army ants or termites digging tunnels in mounds. Yet other groups, like fireflies synchronously flashing, exhibit a very high degree of temporal coherence. It is crucial stressing the fact that all these self-organizing behaviors do not require any external directing influence, but instead solely emerge from repeated local interactions between swarming agents. Understanding these phenomena is a

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central endeavor at the interface of many scientific disciplines: biology, medicine, social sciences, neuroscience, to name just a few. However, throughout this book, the mechanistic aspects of collective behavior—how they are achieved—by which dynamic patterns and responses are developed and maintained are of special interest to us. More emphasis will therefore be put on these behavioral mechanisms over functional considerations—what the benefits of such group behaviors are, while always keeping in mind that mechanisms and functions are in general intertwined. This simplifying assumption is made so as to be able to establish design principles and guidelines being as general and universal as possible. Let us recall that the central focus of this book is the design and control of artificial swarming behaviors. With this goal in mind, collective animal behaviors provide a wealth of interesting case studies amenable to theoretical and numerical modeling, in other words, an inspirational gold mine.

2.2 Ethology

Ethology is commonly defined as the scientific and objective study of animal behavior, usually with a focus on behavior under natural conditions. In general, ethologists tend to favor the study of the behavioral process over an in-depth analysis of precise behavioral traits of a given species, which falls primarily into organismal biology. Given the significance of social animal behavior in nature, a large body of work in the field of ethology is dedicated to the study of collective animal behavior. These collective behaviors typically are the outcome of a suite of interactions that occur between two or more individual animals, usually conspecifics. However, at this point, it is worth distinguishing between two distinct kinds of collective animal behaviors of interest to the ethologist.

First, there are population-level phenomena whose emergence is controlled by changes in intrinsic and extrinsic factors related to ecological factors and/or evolutionary biology. For this first type of collective phenomena, the dynamics and structure of the associated social networks have far-reaching implications for the ecology and biological evolution of individuals, populations, and species [2]. For instance, the study of those phenomena allows us to understand the ability (or therefore lack of it) to adapt to environmental changes of some thriving species while others go extinct. Typically, such phenomena occur over fairly long dynamics—ranging from months to decades, or even longer periods of time for evolutionary processes—compared to the dynamics of typical behaviors occurring on a typical day.

The second class of collective phenomena, which is at the core of this book, concerns group-living behaviors such as those described at the beginning of this chapter and corresponding to short dynamics (up to a couple of days for termites digging ventilation tunnels for the mound) to extremely short dynamics (subsecond to seconds) such as in the case of a school of fish performing evasive maneuvers to escape from a predator's attack. Specifically, our primary interest lies with the dynamical aspects related to: (i) reconfiguration and response, (ii) collective information

transfer, (iii) individual control laws governing the dynamics of the collective, and (iv) fast-distributed decision-making processes. There is no doubt that both classes of phenomena are intertwined: effective short-time group dynamics do provide long-term evolutionary advantages. However, in this book, we will focus our discussion on the short time, and more specifically the extremely short-time dynamic response of animal and artificial collectives, without investigating their implications at the ecological or evolutionary levels.

2.3 Why Biological Inspiration?

In this golden era of engineering design and design science, nobody would dare questioning the importance of biological inspiration. That process consists in using principles from biology to generate novel designs through integration with the most innovative human engineering. These design principles have inspired countless new designs, from new manufacturing processes, to control circuits, flying objects, self-cleaning dry adhesives, and autonomous swarm of robotic platforms. Having said that, it appears clearly that biological inspiration carries a particular significance when it comes to designing swarming systems. It is actually only fair to say that the whole notion of swarm originated from the observation of some biological systems from the animal kingdom that exhibit patterns of collective action. It is only very recently, with the realization of the importance of complex systems, that researchers considered developing artificial designs of swarms, whether in the form of algorithms or with actual agents interacting and evolving in the physical world.

Biology, and its sub-discipline ethology, are not just important because they have contributed to raising our awareness of the pervasiveness of swarms and of the considerable prospects they offer. The accumulation of empirical knowledge of animal group behavior across species is monumental owing to the dedicated efforts of scores of ethologists. This wealth of information on collective animal behavior contributes to improving our understanding of the fundamental elements underpinning the dynamics of swarms in natural systems. Among these elements are the evolutionary optimized mechanistic components responsible for all emergent collective behaviors. A detailed account of such mechanisms clearly offers an invaluable source of inspiration. Comparatively, all the scholarly research carried out to gain insight into the functional considerations of swarming, and the associated suspected benefits for the animal group, are probably of less interest to the swarm designer—unless the final objective is to devise an artificial swam that is solely limited to copying its natural counterpart. That would amount to pure biomimicry.

However, the serious difficulties faced by ethologists when trying to relate collective behavioral function and the underlying mechanisms at the individual level is revealing of one of the key challenges in swarming design: namely, to find the appropriate set of local rules, which through repeated applications, in specific environmental conditions and with appropriate agent characteristics, will lead to a desired

collective behavior of interest. In itself, this stiff problem can be considered to fall in the class of inverse problems, and there is no doubt that it is a wicked problem.

As highlighted at the beginning of this chapter, collective animal behaviors represent an inspirational gold mine for designers and developers of swarming systems. For instance, insects colonies have been an amazing source of inspiration for the development of novel optimization and distributed problem solving techniques. Ants are capable of solving stiff problems far beyond the means of each individual. However, most collective tasks for which these swarming systems are developed—e.g., search and rescue operations, distributed surveillance or detection, delivery systems, etc.—are not generally performed by natural swarms, or at least not exactly in the way that the designed artificial system is intended to operate. It is therefore instrumental not being overreliant on or fixated with swarming solutions developed through millennia of evolution. This word of caution is also meant to stimulate more nonbiologically driven experiments with natural swarms. Indeed, ethologists go at great length to develop experimental setups that reproduce environmental conditions of natural swarms as faithfully as possible. Among the many difficulties faced by ethologists, the identification and tracking of every individual swarming agents is probably the most challenging, but it is required in order to understand the cohesive and dynamic response of collectives. Only with that, an improved understanding of the functions of swarming can be achieved. However, some researchers have started to carry out experiments in which collective animal behaviors arise in conditions rarely (or even never) encountered in the natural world. A good example of such experiments, would consist in constraining the schooling behavior of a collective of fish to a two-dimensional environment, e.g., by using a very shallow tank thereby preventing the three-dimensional roaming of individual school members. Despite their lack of biological relevance, these experiments could help gain new insights into the functioning of swarming systems. Not only these experiments could prove to be very valuable in that respect, but they are also probably less demanding in terms of experimental resources and could therefore more easily be carried out.

All the arguments listed above provide an overall positive answer to the question formulated in this section's title: "Why biological inspiration?" Nonetheless, one should keep in mind an important point emphasized throughout this book: the design of swarming systems is a multidisciplinary endeavor, and consequently, sources of inspiration should be sought in all fields that could potentially contribute to the successful development of a swarm. The following chapters are intended to contribute to that goal.

2.4 What Nature Teaches Us About Swarming

In this section, we will briefly review some of the central tenets of swarming that have been established through decades of rigorous and painstaking observation, experimentation, analysis, and modeling of collective animal behaviors. Of course, this

review is incomplete and readers seeking an exhaustive account should consult monographs by experts specialized on this topic, e.g., Refs. [3–5]

2.4.1 Self-Organization and the Importance of Order in Life

Quite interestingly, one can say that without self-organization and swarming there would be no Life as we know it. Indeed, Life's order is characterized by a cascade of emergent phenomena; emergence being defined as the spontaneous self-organization of a system made of interacting internal agents, without intervention by external directing influences [5]. This fact was acknowledged long ago by the physicist and Nobel laureate, Erwin Schrödinger, in his distinguished monograph "What is Life" [6]. Therein, Schrödinger stressed the challenges faced by the physicist and the chemist in apprehending some of the complexities encountered in life sciences. Some of these challenges will be reviewed in Chap. 3, where the emphasis is put on collective behaviors of inanimate agents. As already mentioned and also shown in Fig. 2.1, self-organization in biological systems pervades nature and takes a central part into the morphogenesis of the vast majority of multicellular living organisms. This was recognized by Macklem [7] as one of the two secrets in Life based on Schrödinger's treatise.

To identify the central tenets of swarming, one needs to consider each and every ingredient required to produce order on a large scale, i.e., a pattern. To make matters worse and as we will see below, this order may not always be apparent. For static or dynamic patterns of self-organization to emerge, the system's components—e.g., cells, amoebae, fish, birds, or swarming agents—must intercommunicate, interact, and cooperate; these communications and interactions being typically local in the natural world. Hence deciphering emergence in a complex biological system requires a clear understanding of the following elements:

- *Interactions among agents*. They can be divided into two categories: physical and trophic. Purely physical interactions—i.e., involving any of the fundamental laws of physics, e.g., mechanical or thermal—are responsible for collective phenomena involving inanimate agents. Trophic interactions involve material flows that have specific effects on the metabolism of the recipient—typically energy, nutrients, repellents or toxins.
- *Informational exchanges*. These could be considered as a third category of interaction. In this book, we would like to draw a clear line between actual interactions and informational exchanges as the latter largely prevails in artificially swarming systems. These informational exchanges are either unidirectional or bidirectional and involve one or more sensory modalities.
- *Information processing*. The information externally acquired through the sensing of the environment and the detection of other conspecifics is internally processed through a complex and still-largely-unknown cascade of cognitive processes varying largely from taxon to taxon. Oversimplifying grossly, this process can be

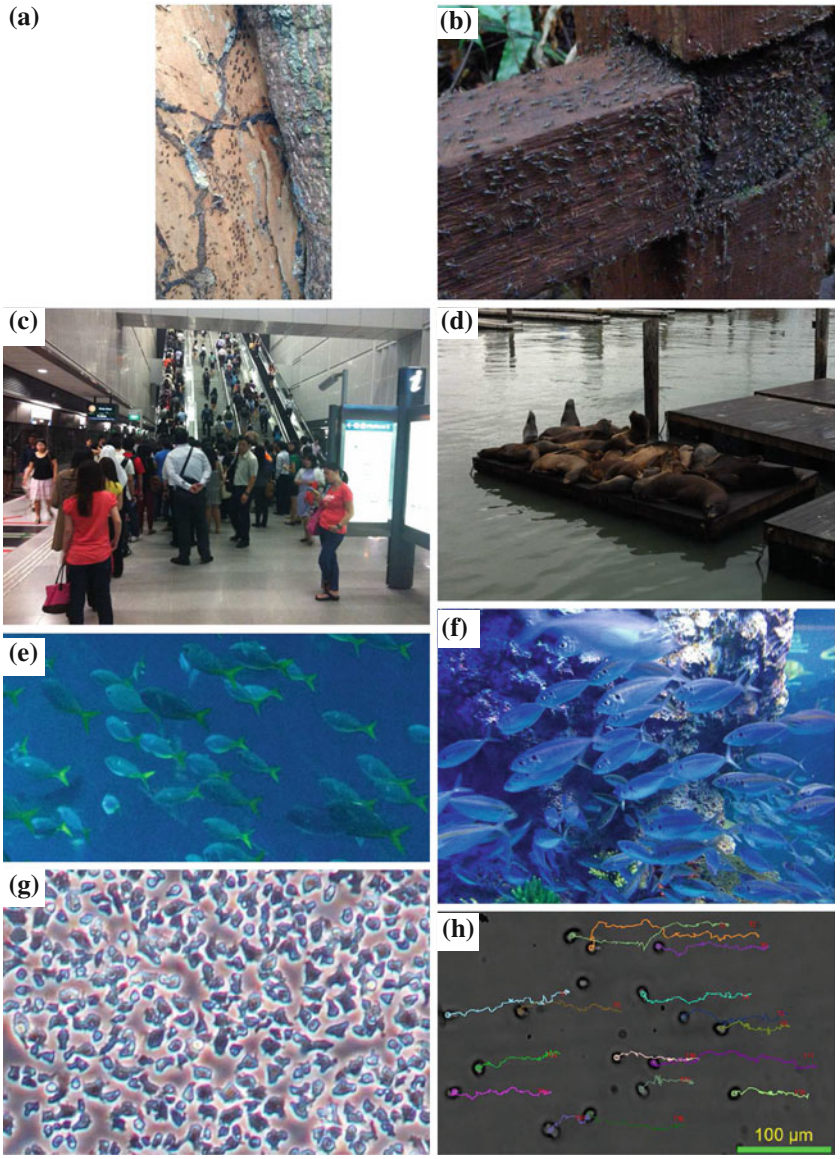

Fig. 2.1 **a** Lane of aphids collectively moving along a tree trunk (Upper Seletar Reservoir, Singapore); **b** Swarm of unidentified insects (Lower Peirce Reservoir, Singapore); **c** Lane formation in human crowds (Mass Rapid Transit station, Singapore); **d** Collective napping of sea lions (San Francisco, CA); **e** School of yellowtail fish (Pulau Besar, Malaysia); **f** School of unidentified fish (S.E.A. Aquarium, Singapore); **g** Clustered starved *Dictyostelium discoideum* amoebae (Bouffanais' lab, SUTD); **h** Mechanotactically induced collective migration of *Dictyostelium discoideum* amoebae (Bouffanais' lab, SUTD)

reduced to a filtering and integration signal processing during which salient features of the externally acquired information are extracted—e.g., sudden changes in the direction of travel are swiftly and accurately picked up by birds in a flock or fish in a school.

- *Behavioral algorithm and response*. The externally acquired and subsequently processed information is used by the animal to shape a behavioral response according to the specifics of the internally stored genetic information.

All of the above is very well summarized by Sumpter [8], who argues that the key to understanding collective animal behaviors—and more broadly the concept of self-organization and the spontaneous emergence of order—lies in identifying the principles of the behavioral algorithms followed by individual animals and how information flows between the animals.

As a matter of fact, the next chapters deal specifically with individual elements listed above. However, as is common with complex biological systems, a clear distinction between each of these elements is not always possible as they can be simultaneous and/or interwoven.

2.4.2 Positive Feedback and the Emergence of Order

Regardless of the particular level of development, the animal kingdom is filled with swarming behaviors. Among these swarming behaviors, aggregation patterns are almost universal across living organisms, from bacteria to higher vertebrates. It is now well known that these aggregation patterns as well as other forms of collective behaviors use various forms of positive feedback mechanisms [5].

A celebrated example of the importance of such positive feedback mechanism is the chemotactic aggregation of the social amoebae *Dictyostelium discoideum* (Dd). This simple eukaryote is a genetically, biochemically, and cell-biologically tractable model organism [9], which has been a microorganism of choice for studying a variety of basic processes in morphogenesis, including cell–cell chemical signaling, signal transduction, and cell motility. Of particular interest to us is the extensively studied social life [10, 11] of this prototypical motile cell. Dd exhibits a remarkable interplay between single-cellular and multicellular behaviors. Specifically, in the presence of nutrients, individual Dd cells move, feed, and divide every few hours. Food scarcity leads to a behavioral change in Dd toward a collective aggregation, which leads to a variety of wonderful complex spatial patterns. Although a complete understanding of the exact mechanisms at play during such collective aggregation remains elusive, it is generally agreed that a specific chemical, namely cyclic adenosine 3', 5'-monophosphate (cAMP) plays a central role in the orchestration of the aggregation process. This cAMP is secreted by the cells and used for intercellular communication. Without delving into the details, it has been revealed experimentally that when Dd cell sense and detect cAMP released by neighboring cells, it triggers the secretion of cAMP by the cell. In turn, this reinforces the chemical signal in the

extracellular environment thereby increasing the overall signaling range necessary for the collective aggregation to perdure and for order to emerge on very large scales as observed in aggregation patterns.

In the engineering world, positive feedback loops induce amplification typically leading to a destabilization of the system. The triggering of stabilizing negative feedback mechanisms may at some point counterbalance these destabilizing effects thereby leading to a new dynamics of the system. In the example of the aggregating amoebae, some form of "recycling" of cAMP occurs to avoid a saturation in this chemical within the environment. Without cAMP recycling, cells would lose their ability to sense the chemical and detect gradients of cAMP, which would totally inhibit the aggregation process. For other collective behaviors built upon other forms of positive feedback, the counterbalancing effects of negative feedback may come from the induced depletion in resources required by the exponential growth of the system. Finally, for collective behaviors of the consensus type—a.k.a. imitative behavior such has herding, flocking schooling, synchronous flashing or rhythmic applause, etc.—negative feedback mechanisms may not be as clearly and easily identifiable.

In summary, positive feedback is a powerful mechanism to create structure and induce coherence (in time and/or space) in collective phenomena in general, and in self-organizing biological systems in particular.

2.4.3 Collective Behavior Without Large-Scale Order

All too often, collective behaviors are mistakenly associated with apparent collective order in space or collective synchronization in time. Indeed, biologists and nonexperts alike can readily detect large-scale organizations in schools of fish and flocks of birds, and also the high level of temporal coherence in the synchronous flashing of fireflies or even the rhythmic applause of audiences at the end of concerts. These patterns can be caught by the naked eye or the naked ear. However, statistical physics and the theory of dynamical systems inform us that nonapparent order may develop in interacting many-body systems. This nonapparent order is typically characterized by long-range correlations in fluctuations of the state variables. An interesting case in that respect is that of turbulent flows, which to the untrained eye appear as chaotic and lacking order. But, a fine analysis of such flows reveals the emergence of large-scale Lagrangian coherent structures associated with fluctuations of the velocity field [12]. Note that these vortical coherent structures have spectra governed by specific power laws which are the signature of a certain organization and dynamics in complex systems.

Interestingly, Cavagna and co-authors [13] have observed the occurrence of collective behaviors in swarm of insects—midges—in the absence of large-scale collective ordering. As stressed by Cavagna and his team, it is critical not to identify collective behavior with collective order. Beyond our search for universal principles in swarming, these important observations reported in Ref. [13] remind us about the vast diversity in possible swarming behaviors. The comparison between birds and flying

insects thereby provides an interesting perspective. It is clear that differences in: (i) sensory modalities—amounting to differences in how information is received by each individual agent, (ii) cognitive abilities—how information is internally processed by the neural system, and (iii) behavioral response—how the agent responds to the information acquired and processed, are responsible for the emergence, or not, of large-scale order. Specifically, birds in a flock interact topologically while midges interact according to a metric distance. Moreover, the mobility of midges is far more "erratic" as compared to starlings for instance. Another interesting example with microorganisms is the chemotactic aggregation of bacteria as compared to the one of amoebae. Bacteria such as *E. coli* for instance are indirectly guided organisms [14], performing a biased random walk—through the now famous run-and-tumble kind of motility [15]—in the direction of the gradient of the chemotactic signal. Amoebae such as *Dictyostelium discoideum* are directly guided eukaryotes [16] that are able to climb fairly shallow chemical gradients with a remarkable accuracy [11]. Given such differences in sensing, processing and responses exhibited by *E. coli* and *D. discoideum*, it is therefore not surprising that their respective chemotactic aggregations are noticeably different.

We would like to return to the study of Midges in the field carried out by Cavagna and his team [13] as it calls for a new appreciation of what a swarm really is. According to Cavagna et al., the true hallmark of collective behavior in animal systems is correlation rather than order. We strongly back this view as it is becoming apparent that a too intense ordering of a swarm seems to hinder its ability to swiftly respond to fast condition changes in its surroundings. Interestingly, the new light shed on the pivotal role of high correlations in collective behaviors recalls a hallmark of complex systems, which are known to have a dynamics at the edge of chaos [17]. By that it is meant that its degree of organization lies somewhere between complete order and utter chaos. This last point will be revisited in Chap. 3.

Finally, it is worth adding that until now, this important realization about the importance of high correlations over high ordering has not been considered as a key guiding principle for the design of swarms.

2.4.4 Information Processing and Swarm Intelligence

The vital role played by information processing in the dynamics of living systems has been long acknowledged. Four decades ago, Gatlin already wrote that "Life may be defined operationally as an information processing system—a structural hierarchy of functioning units—that has acquired through evolution the ability to store and process the information necessary for its own accurate reproduction" [18]. Furthermore, information processing or equivalently computation is now known to be at the core of the dynamics of complex adaptive systems. It is therefore not surprising to find information processing as one of the central tenet of the dynamics of swarms, which are complex systems made of living units.

As was seen in the previous section, large-scale ordering may not be a true hallmark of collective behavior but it is becoming clear that large-scale information processing is one. This is readily observed in the optimal and adaptive foraging behavior of ant colonies as well as their ability to dynamically adapt to changing environmental circumstances through a fully decentralized task allocation process. Another very interesting and important example is the collective response of swarms of mobile agents exhibiting surprisingly fast responses such as dramatic stampedes in high-density human crowds, flash expansion by school of fish and collective turn by flock of birds when confronted by predators attacking. All these examples confirm the fact that information exchanges between individuals go hand in hand with some form of positive feedback: one agent imitates a neighbor's behavior, which in turn induces an imitation of other agents in its neighborhood, and thus a cascading effect typical of positive feedback ensues.

Probably, the most important difference between collective behaviors of inanimate agents (see Chap. 3) and swarms of living units resides in the ability of the latter to collectively process and respond to information in the form of signals and stimuli. There is no doubt that it contributes to the greater complexity and variety of collective behaviors observed in the natural world as compared to those encountered in the physical world. The term "swarm intelligence" is colloquially used to refer to such emergent and adaptive collective responses of groups of simple agents [19]. The past two decades have experienced substantial work aimed at mimicking specific collective behaviors of social insects. The reason for this unabated interest comes from the fact that some insects can solve stiff problems in a very flexible and robust way. Termites showcase flexibility as they are able to mend any damaged part of their mound with the same remarkable effectiveness. When ants, bees, and termites perform their unique distributed problem solving skills, they display high levels of robustness as they continue to function despite the possible loss of a large fraction of individual members. All of this with each individual insect following a very simple set of local rules and without any knowledge about the overall swarm and pattern in motion.

Robustness and flexibility are very appealing features for an engineering system. From the design standpoint, the above important facts suggest that information processing and control (i.e., the behavioral response) will be central elements to be considered and integrated into any innovative design of a swarming system.

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Chapter 3
A Physical Approach to Swarming

The previous chapter was centered on awe-inspiring collective behaviors observed in the animal kingdom, and how this inspirational bounty can be harnessed to develop innovative swarm designs. We stressed two important points related to some very common collective behaviors: (1) they occur across vastly different spatial scales—from the microscale world to our macroscale world, and (2) they emerge across immensely different taxa—from bacteria to quadrupeds. When faced with such empirical evidences, the physicist will immediately suggest the existence of universal mechanisms at the root of collective phenomena that would justify and explain such commonalities in dynamic behaviors. This may appear counterintuitive at first since the fundamental laws of physics yield forces and energies of very different magnitudes across a wide range of scales. For instance, swimming at the micrometer scale requires aggregating microorganisms—e.g. bacteria or amoebae—to power themselves by harnessing viscous forces which form the main source of drag for macroscale schooling fish. The undeniable successes of physics in the past centuries certainly come from an uninterrupted search for universality across seemingly unrelated phenomena—e.g. dispersion in optical waves and dispersion in mechanical waves. It comes therefore with no surprise that physicists have been extremely active, and successful, in the past two decades uncovering the prominent universal mechanisms at play in collective phenomena at large. This chapter will present a brief overview of those while stressing their importance for the designer of swarming systems.

3.1 Self-Organization in Physicochemical Systems

As mentioned in Chap. 2, under specific circumstances, groups of locally interacting units give rise to emergent collective behaviors characterized by complex adaptive patterns. The underlying process of self-organization is pivotal and occurs spontaneously through repeated interactions (primary at the local level) internal to the system and without the action of a global driving or directing influence. This definition is

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repeated here to underscore that any kind of system—be it physical, chemical, artificial, social, or even virtual—and not just biological ones, may exhibit swarming-like behaviors.

This reminder is justified by the persistent, although incorrect, perception that only biological systems or social systems are true complex systems exhibiting emergent behaviors. First and strictly speaking, it is more appropriate to talk about a complex behavior rather than a complex system [1]. This subtle point emphasizes that the dynamic adaptive response of such collectives is complex although the individual interacting and communicating components may be extremely simple. Of course, in natural swarms the interacting agents have their own individual complexity. This complexity at the agent level does contribute to an increased variety in the achievable complex self-organizing behaviors. However, it certainly is not a prerequisite for generating them as we will see in the coming examples.

3.1.1 Elementary Cellular Automata

The cellular automata (CA) approach is a powerful method to describe, model, understand, and simulate dynamic collective behaviors. This approach is particularly useful owing to its relative apparent simplicity and also given the limited choices of elementary models available to study nonequilibrium statistical physics problems. CA probably offer the simplest minimalist framework for a system dynamics composed of locally interacting agents. These agents—cells as they are called in the CA lexicon—are as simple as can be with a binary state space (0/1, on/off, black/white) [2]. Despite all this built-in simplicity, CA exhibit an incredibly wide variety of complex behaviors.

The objective here is not to give a lengthy introduction about CA (which can be found in the following treatises, Refs. [2, 3]) but instead to present some insightful examples of emergence of specific patterns in a system made of extremely simple nonliving agents subjected to equally simple interacting rules. In its minimalist form, CA are an idealization of a physical system in which space—a one-dimensional one to simplify as much as possible—and time are discrete while the state variable takes binary values. The cells are regularly arranged along a one-dimensional chain or lattice (see Fig. 3.1), which can be rendered infinite by means of periodic boundary conditions. To keep the system as simple and tractable as possible, we consider the simplest possible interaction between cells, namely with its direct neighbors to the left and right sides. Wolfram has systematically studied all the $2^8 = 256$ possible rules of interaction and extensively documented their dynamics [2]. Given a rule and an initial state for each cell, one can deterministically compute the ensuing dynamics of the system, as shown for 4 distinct rules in Fig. 3.1. Wolfram has shown that all 256 elementary CA rules can be categorized into 4 groups associated with the type of dynamical behavior exhibited. Among these 4 groups, one is characterized by the emergence of persistent complex structures for most of the initial conditions considered. The CA rule 110 falls into this group and is explicitly defined by

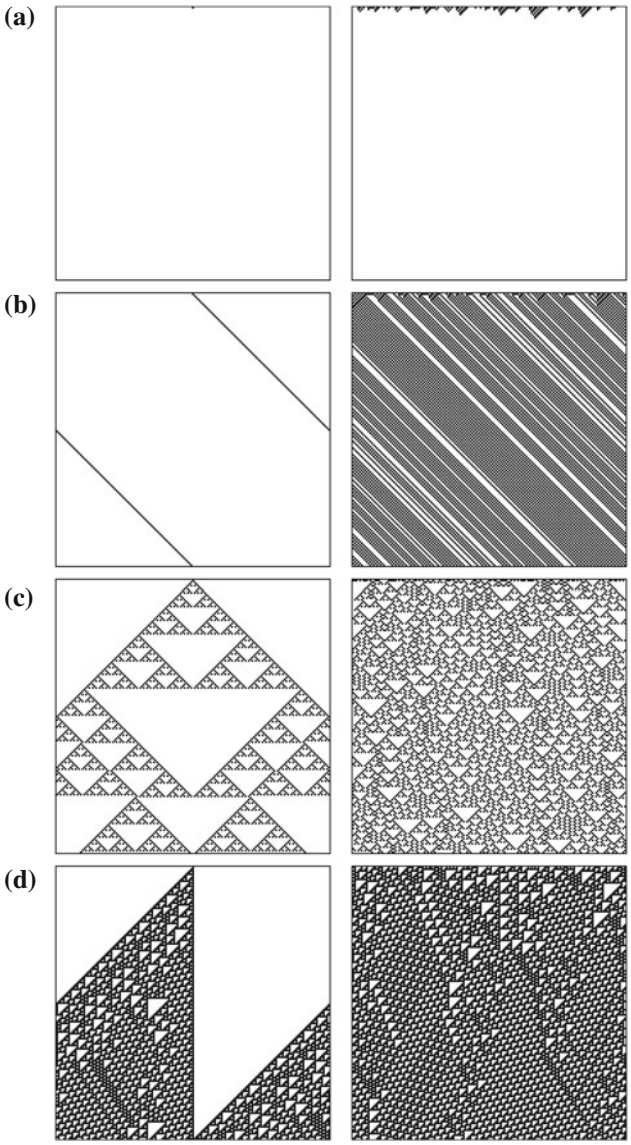
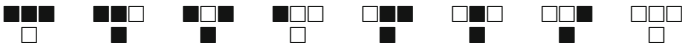


Fig. 3.1 Time evolution over 160 timesteps for 4 distinct elementary cellular automata rules according to Wolfram's definition [2]. In each graph, the *top row* represents the initial condition, which is either just the middle cell "on" (■) and all others "off" (□) on the *left column*, and a random distribution of states on the *right column*. Periodic boundary conditions, giving the lattice a torus-like topology, are enforced: **a** Rule 40 that belongs to the category of CA converging to a limit or fixed point in the phase space; **b** Rule 56 that belongs to the category of CA converging toward a stable configuration equivalent to a limit cycle; **c** Rule 18 that belongs to the category of CA generating a chaotic outcome, such that a small change in the initial condition almost always yields vastly different outcomes with fractal-like self-similar patterns and showing a dynamics analogous to those observed in the presence of strange attractors; **d** Rule 110 which has been proved to be Turing complete



where  means that if the cell of interest (in the center) is in the state □, while its right neighbor is in the state □ and its left neighbor is in the state ■, then at the next timestep, this center cell will remain in the same state □. Two examples of dynamics of rule 110 are shown in Fig. 3.1d. Rule 110 is a well-celebrated rule as it has been proved to be Turing complete—i.e., capable of universal computation, see Chap. 6—and its behavior cannot be predicted and can be determined only by explicit simulation of its dynamics.

In summary, CA represent a natural modeling framework to describe and study physical systems comprised locally interacting components. Beyond this important fact, this brief descriptive introduction to CA allows us to develop an intuition about the theories of physics that are central to swarming:

- *Complexity Theory*: Basic nonliving agents subjected to very simple local interaction rules can yield complex dynamic behaviors: read biological complexity is not a prerequisite to emergent complex dynamics.
- *Theory of Dynamical Systems*: The local coupling of many simple dynamical systems gives rise to global behaviors with dynamics leading to fixed points (e.g. rule 40), limit cycles (e.g. rule 56), chaos (e.g. rule 18), or more interestingly, dynamics "at the edge of chaos" (e.g. rule 110). This latter expression will be further explained and elaborated in the sections below.
- *Statistical Physics*: CA are based on microscale idealizations of physical phenomena. It is a well-established fact in statistical physics that many macroscopic behaviors of particle systems depend minimally on the details of the microscopic interactions between the elementary constituents.

3.1.2 Collective Phenomena in Physical Systems

From the conceptual standpoint, the examples of various dynamics manifested by different elementary CA rules shown in the previous section were eye opening. However fascinating they are, CA remain an idealization rooted in the simulation world. Nonetheless, numerous manifestations of self-organization in purely physical and chemical systems exist. As already emphasized in Chap. 2, self-organization in collectives of inanimate objects is solely based on physical interactions among system components, thereby excluding trophic interactions and informational exchanges as detailed in Sect. 2.4.1. Three paradigmatic examples in both chemical and physical systems are given below.

First, we consider the emergence of convection cells—so-called Bénard cells—of particular shape in a thin layer of a viscous fluid having specific surface tension properties. These Bénard cells spontaneously form when the temperature difference across the layer of fluid crosses a critical threshold value. At that critical temperature

difference, a bifurcation takes place and long-range correlations, as compared to the short range of intermolecular forces, reveal the large-scale organization of the fluid system.

Our second example is taken from the chemical world and is the thoroughly studied Belousov–Zhabotinsky (BZ) reaction. This reaction is classically presented as an example of nonequilibrium thermodynamics, associated with the emergence of a nonlinear chemical oscillator. As a consequence, BZ reactions are far from equilibrium and remain so for a significant length of time and may in some circumstances evolve chaotically. Self-organization in BZ reactions is manifested by the spontaneous development of eye-catching patterns: e.g. target patterns, spiral waves, and multiarmed spirals [1].

The third and last paradigmatic example is related to the natural and spontaneous rippling of sand dunes under the combined effects of gravity and wind. At a simpler level, the local process of accumulation of sand along with the formation of a sandpile has attracted a significant interest from the physics community. It is easy to develop an intuition of the dynamics of that process, especially if you have experimented with it during your childhood days at the sea side. It starts with an almost flat sandpile. With the continuous addition of sand—dropped from an unchanged location—the pile becomes steeper with the possibility of some minor sand slides. As the process continues unfolding, these sand slides become more and more important. Eventually, some of these sand slides occur over length scales equivalent to the size of the pile itself. From that point onward, the system is in a critical nonequilibrium state such that the correlation length of the system and the correlation time of the system go to infinity, without the need to vary any control parameter [5]. This contrasts with examples of critical phenomena encountered in equilibrium thermodynamics, such as the phase transitions between solid and liquid, or liquid and gas, where the critical point can only be reached by precise changes of the temperature—the control parameter. In this sandpile problem, we are in the presence of self-organized criticality [5]. Many experiments of the heaping in a sandpile have been reported in the literature with varying setups and conditions. More in-depth and quantitative analyses of sandpile dynamics have been achieved using numerical models based on CA, e.g. the two-dimensional Abelian sandpile model, also known as the Bak–Tang–Wiesenfeld model [4]. With this latter model, we are able to observe fascinating patterns emerging in the self-organized critical state, see Fig. 3.2 (left). These CA-based models uncovered the fact that once the sandpile model reaches its self-organized critical state, the system's response becomes decorrelated from the actual continuous dropping of sand grains. Specifically, dropping another grain of sand onto the pile may cause nothing to happen, or it may cause the entire pile to collapse in a massive slide. A more realistic sandpile model involving the random dropping of sand grains [5] allows us to clearly observe the transition a the self-organized critical state (see Fig. 3.2 (right) after approximately 1,000 sand grains have been dropped). One of the features of this model that has attracted much attention is the appearance of a $1/f$ noise in the self-organized critical state. This feature is common to many complex

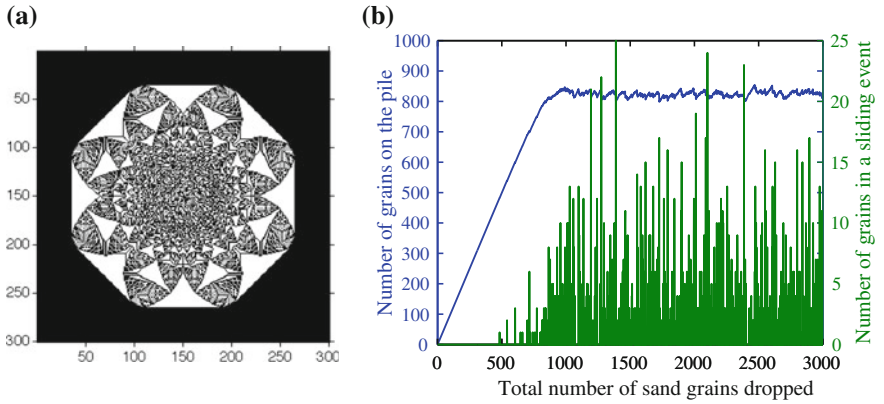


Fig. 3.2 **a** Sandpile pattern obtained with the Bak–Tang–Wiesenfeld model [4] after 60,000 sand grains were dropped at the center of a 300 × 300 square lattice. Notice the fractal-like subpatterns. **b** Emergence of self-organized criticality in sandpile formation following the Bak–Paczuski model [5] in which sand grains are dropped at random locations. After dropping approximately 1,000 sand grains, the total number of sand grains on the pile stops climbing and instead starts fluctuating. At that point, the system has reached a self-organized critical state characterized by sliding events in $1/f$ where f is the frequency of the event

systems and has been observed in situations as diverse as the flow of the river Nile, light from quasars, and many other natural systems [5].

3.1.3 Collective Motion

The above three examples, although compelling, are of limited direct relevance to our subject of interest: swarms of mobile interacting agents. However, in the last decade, there has been mounting interest in experimental and theoretical studies of this particular kind of collective phenomena, namely collective motion. This surge in interest follows three decades of very productive and successful research based on self-propelled particle models [6, 7] and cellular automata [8] to name a few. Given how closely related these phenomena are to natural swarms, we will provide a short list of the most notorious works and refer the interested reader to the recent comprehensive review by Vicsek and Zafeiris [9].

Using experiments involving self-propelled anisotropic vibrated rods numbering in the hundreds to a thousand, Kudrolli et al. have shown the important role played by the shape of the rod in the collective dynamics of the mobile rods exhibiting local ordering and aggregation at the side walls [10]. With the same self-propelled rods, Kudrolli further studied the collective dynamics at higher density of agents, close to the packing limit [11].

Working with a fluidized monolayer of macroscopic rods, Narayan et al. revealed nonequilibrium steady states with tetratic, nematic, and smectic correlations [12]. When the rods were in the nematic liquid crystalline phase, Narayan et al. studied their collective dynamics and reported long-lived fluctuations featuring giant number fluctuations (GNF) [13]. Their results were significant as they revealed that swarming behaviors can appear in a system in which units do not communicate except by contact. In the same vein, Schaller et al. have demonstrated the emergence of collective motion in a high-density motility assay comprising actin filaments driven by molecular motors with the filaments motion being confined to two dimensions [14]. Beyond a certain critical threshold in the filaments density, the actin filaments self-organized forming coherently moving patterns exhibiting fluctuations with properties similar to those observed by Narayan et al. [13].

At the microscale, Ibele et al. have developed micrometer-sized and light-powered motors made of silver chloride (AgCl) particles that move in deionized water by means of self-diffusiophoresis [15]. This very innovative microswimmer design enabled Ibele and his co-authors to generate and study swarming behaviors based on a nonbiological model for intercellular communication reminiscent of the chemotactic aggregation of *Dictyostelium discoideum* amoebae (see Sect. 2.4.2). Still at the microscale, Bartolo and his team have recently achieved collective behaviors of micrometer-sized motile colloids [16]. This work is significant for several reasons. First, the driving mechanism used to power the motion of the colloid beads is extremely ingenuous and is based on an overlooked electrohydrodynamic phenomenon known as the Quincke rotation. Second, the colloid particles are afforded short-range sensing capabilities, giving them information about the orientation of their neighbors, by means of a combination of hydrodynamic and electrostatic mechanisms. Lastly, Bartolo and his team are the first to generate such artificial collective motions with a swarm consisting of millions of units. This achievement allows for the first time to experimentally investigate universality and scaling behaviors in such nonequilibrium statistical physics systems.

Finally at the macroscale, many swarms of robotic platforms have been reported. For the sake of the discussion and without trying to provide an exhaustive overview of the state of the art, we highlight two recent studies emphasizing swarming aspects over the robotics ones. Suematsu et al. have studied a swarm of camphor boats placed in an annular water channel. The interaction between the surface vehicles is obtained through the concentration of the camphor molecules in the water [17]. With this original setup, Suematsu and his co-authors generated collective behaviors bearing similarities with dynamics encountered in traffic flow and ant trails. Recently, Rubenstein et al. reported the largest swarm of macroscale robots made up of a thousand units—the now famous "kilobot"—achieving programmable self-assembly through local interactions [18].

From the above overview of the state of the art, two central issues are arising. First, similarly to what is observed with animal swarms (see Sect. 2.4.2), swarms of inanimate objects solely based on physical interactions can be produced across vastly different length scales, from the microscale world [10–16] to our macroscopic realm [17, 18]. As mentioned at the beginning of the chapter, this fact reinforces

our intuition that at the core of swarming behaviors, some universal mechanisms are at play. Second, these universal mechanisms can be said to be rooted in the following theories of physics: (i) statistical physics: density effects, phase transitions, and scaling laws, nonequilibrium states with large-scale correlations and GNF; and (ii) theory of dynamical systems: bifurcation, symmetry breaking, self-organization, avalanches, critical states, and self-organized criticality. In the sequel, we will discuss some aspects of these theories that are relevant to swarming dynamics.

3.2 The Self-Propelled Particles (SPP) Model

Throughout this book, we will use and refer to what probably is the most widely used and celebrated model of swarming, namely the self-propelled particles (SPP) model. This model was introduced in different forms by Aoki [7] in his early studies of fish schooling, by Reynolds [19] in the field of computer graphics for applications in video animations of dynamic herds of wildebeest, followed by the seminal work by Vicsek et al. [6]. The SPP family of models has received a tremendous amount of attention from ethologists, physicists, social scientists, and engineers alike. It is not our intention to delve into a long review of the very many variations around the SPP model. The interested reader may consult the comprehensive review by Vicsek and Zafeiris [9]. Instead, our intention is to introduce one relatively classical version of the SPP model—a vanilla flavor of some sort— which we will continue using in the following chapters, and that will serve us to illustrate and better comprehend many abstract theoretical concepts.

3.2.1 Dynamical Foundations

We consider a collective of N locally interacting adaptive and identical individuals. Each individual agent i, at any given instant t, is assumed to be fully characterized by the state variable $\psi_i(t)$. Such a generic state variable may represent widely different characteristics depending on the nature of the group considered: e.g. employed or unemployed forager state for honey bees, kinematic variables for fish in a school, birds in a flock or robots in an artificial swarm, space available for a pedestrian on a congested sidewalk, etc.

The nonlinear dynamics of each agent i takes the general form

$$\frac{\mathrm{d}\psi_i(t)}{\mathrm{d}t} = f(\psi_j(t), \psi_{j+1}(t), \ldots, \psi_{j+k-1}(t), \psi_i(t)), \tag{3.1}$$

that stresses the local nature of the interactions between agents since the subset $\Psi_i(t) = \{\psi_m\}_{m=j,\ldots,j+k-1}$ only includes a fraction k of the N agents affecting the behavior of agent i. Note that the formalism of Eq. (3.1) does not capture the fact that

the value of the k indices—from j to $j+k-1$ above—are actually i-dependent since they are defined by the belonging, or not, of an agent to the neighborhood of interaction of agent i. Moreover, these k indices may change over time due to the dynamical nature of the neighborhood of interactions, itself imposed by the dynamics of agent i. That means that in general, the makeup of Ψ_i varies from individual to individual and changes with time. Specifically, it is entirely dependent on how the neighborhood of interactions—formally represented by Ψ_i—is constructed which further defines the communication links between agents. The neighborhood of interactions is the cornerstone of the global swarm signaling network (SSN, see Sect. 4.2), and its intricate structural properties and dynamics have been studied below. Moreover, the values of each ψ_m within Ψ_i are made available to the internal control processing mechanism through the various sensory modalities defining multiple communication channels between group members—e.g. mechanical signaling through lateral line sensing and visual signaling are both involved in fish schooling [20]. The function f in Eq. (3.1) embodies the specifics of each individual's internal control processing mechanism (see Sect. 6.3). It is worth reminding at this stage that complex collective dynamics can be achieved with simple f as was already discussed in Sect. 3.1.1 with elementary CA models and also given the possibly nontrivial dynamics of Ψ_i depending on the very nature of the neighborhood of interactions.

At this point, we make another general assumption consisting in imposing that any decision made by a group member is based on relative state values and not on absolute ones. If the state variable ψ_i is a quantity that is frame dependent, such as the agent's velocity, the agent is solely able to appreciate an interacting neighbor's state with respect to its own. This argument may even hold for nonframe-dependent state variables—e.g. pheromone levels in ant trails—and is easily reconcilable with the multiple gradient-based taxes observed in many biological systems [21]. Formally, this relative state assumption reads

$$\frac{\mathrm{d}\psi_i(t)}{\mathrm{d}t} = g(\psi_j(t) - \psi_i(t), \ldots, \psi_{j+k-1}(t) - \psi_i(t)). \tag{3.2}$$

The function g is referred to as a consensus protocol—intrinsically local by the nature of its inputs $\tilde{\Psi}_i(t) = \{\psi_m - \psi_i\}_{m=j,\ldots,j+k-1}$—if a steady state can be reached and once it is reached, if the following relations hold: there exists a function h such that

$$\psi_i(t) = \cdots = \psi_N(t) = h(\psi_i(0), \ldots, \psi_N(0)), \tag{3.3}$$

where $\psi_i(0), \ldots, \psi_N(0)$ are agents' initial state conditions. In simple words, the local synchronization protocol defines for each individual agent what Sumpter [22] calls the behavioral algorithm, also known as the internal information processing mechanism responsible for the behavioral's response to the sensed external information that is flowing in a decentralized way throughout the swarm (see Sect. 5.3).

3.2.2 Neighborhood of Interactions

We now aim at formalizing the key concept of neighborhood of interactions. It appears clearly that $\tilde{\Psi}_i$ fundamentally depends on a series of factors which include: signaling mechanisms, sensory, and cognitive capabilities. The signaling mechanisms are the different vehicles for the information to flow through the swarm's surrounding environment. The sensory capabilities are responsible for information acquisition from the surrounding environment to the internal agent domain. Within that domain, the internal information processing is taken care of by the cognitive capabilities (see Sects. 2.4.1 and 2.4.4). Even though the information chain has been clearly identified, we believe that accurately modeling each and every component is nonessential. Indeed, one and only one of those components will be the limiting factor and depending on the environmental conditions that limiting factor may change; e.g. fish schooling from crystal-clear waters to murky ones. This latter point will be thoroughly discussed in Chap. 5.

The basic mechanistic functioning of collective motion is now well understood as being the result of multiple uncoordinated local interactions between individuals. The central importance of these local interactions have led scientists to experiment very many different local interaction rules, often with the aim to reproduce fine details of some of the very specific behaviors associated with different species of swarming agents. However, two broad groups of local interaction rules can be discerned, each based on the definition of a specific interaction distance thereby defining the so-called neighborhood of interaction. The first group based on a metric distance, was the first considered, and has attracted a tremendous amount of attention (see Ref. [9] and references therein). In the metric neighborhood framework (see Fig. 3.3a), each swarming agent exchanges information with all other agents located at a fixed and given distance—assumed to be the same for all [23]. The metric distance was only recently challenged following the analysis of empirical data for the dynamics of flocks of starlings [24] as well as results from the dynamics of human crowds [25, 26]. The European project named Starlings in Flight or STARFLAG has been one of the most recent and largest experiments in the human history carried out to analyze the collective behavior of birds [24]. By reconstructing the three-dimensional positions of individual birds in airborne flocks of a few thousand members, Ballerini et al. showed that the interaction does not depend on the metric distance, as most current models and theories assume, but rather on a topological distance. They discovered that each bird interacts on average with a fixed number of neighbors (six to seven), rather than with all neighbors within a fixed metric distance. To the best of our knowledge, a complete explanation for this surprising empirical observation has yet to be given. Ballerini et al. [24] claim that interactions based on metric distance is unable to reproduce the density changes, typical of bird aggregations, because one would expect cohesion to be lost when mutual distances become too large compared with the interaction range. In addition, with social networks, the relevance of the topological distance between neighbors becomes apparent and it is believed that it could determine how populations move in, split up, and form separate groups [27]. For instance, guppies preferentially shoal with individuals of a similar size [28], and

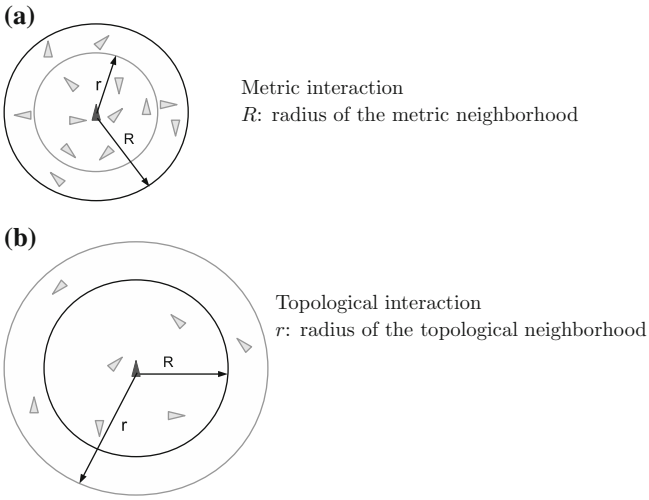


Fig. 3.3 Schematics of metric (**a**) versus topological (**b**) neighborhood of interactions. R is the radius of the metric neighborhood and r is the radius of the topological one based on the rule of k-nearest neighbors with $k = 7$. R is constant as it defines a metric zone around the agent while r changes in accordance with the distance between the agent and its kth (here seventh) nearest neighbor. The topological neighborhood of interaction shows adaptation with respect to changes in the density of swarming agents

faster individuals are more likely to be found at the front of groups [29]. In summary, in the topological distance framework (see Fig. 3.3b), each and every agent interacts with a fixed number of neighbors regardless of the distance separating them.

Essentially, both distances are associated with distinct physiological (resp. technological) limitations of living (resp. artificial) agents. Specifically, the metric neighborhood of interaction finds its origin in the limited sensory range of individuals. Indeed, a fish in a school can only interact with other fish it can perceive either through vision or lateral line sensing [30, 31]. On the other hand, the topological neighborhood of interaction stems from the limited information processing capabilities of individuals. All living or artificial agents possess limited cognitive and information processing capabilities enabling them to socially interact with a fixed number of other agents [21, 32]. However, in real-life situations and depending on their positions within the swarm, individuals may found themselves limited either by their sensory apparatuses or by their internal information processing system. Also, with a topological neighborhood, one has to be watchful for the possibility of the topological distance becoming too large so that the interaction or information exchange could not take place. In practice, that can potentially happen with very low density swarms or when some individual agents become widely separated from the swarm. The rule of k-nearest neighbors epitomizes the topological paradigm. Figure 3.3 illustrates and highlights graphically some of the fundamental differences between a metric- and a topological-based neighborhood of interactions—the rule of k-nearest neighbors is considered. The metric neighborhood or interaction zone

is an open ball with a constant radius, R, centered about the agent while r, the radius of the topological neighborhood, has an adaptive behavior to include the kth (here seventh) nearest neighbor. It is apparent that r is not just a function of the physical distance.

Finally, it is worth adding that a purely metric or purely topological distance is unable to account for this inhomogeneity in limiting factors within the group. This has led some researchers to propose a hybrid interaction distance that integrates both limitations in terms of sensory range as well as information processing [33].

3.2.3 Dynamic Update Rule

In the previous sections, we emphasized the generality of the concepts at the core of the SPP modeling framework. Thus, details such as the nature of the state variable or the type of interaction between agents were intentionally left out. These specific details are not expected to have an impact on the key features at the heart of emergence in collective behaviors; this approach can be regarded as a "crude look at the whole" as advocated by the Physics Nobel Laureate Murray Gell-Mann [34]. It is also very much line with our search for universal mechanisms at the core of swarming behaviors as discussed in Sect. 3.1.3.

We consider self-propelling agents moving about a two-dimensional domain—of dimensions $L \times L$ and rendered infinite by means of periodic boundary conditions—with constant speed, v_0, similarly to Vicsek's model [6]. An important parameter is the density $\rho = N/L^2$ of agents. The neighborhood of interactions can be metric, topological, or anything more complex. For simplicity, we consider a topological interaction distance and further assume that each agent (i) is fully characterized by one unique state variable ψ_i, its velocity $\mathbf{v}_i = v_0 \cos\theta_i \hat{x} + v_0 \sin\theta_i \hat{y}$, or equivalently its velocity direction θ_i, the speed v_0 being constant. The local synchronization protocol, based on relative states and generically stated as in Eq. (3.2), is strictly equivalent to a local alignment rule which mathematically can be stated as:

$$\dot{\theta}_i(t) = \frac{1}{|\mathcal{N}_i(t)|} \sum_{j \in \mathcal{N}_i(t)} w_{ij}(\theta_j(t) - \theta_i(t)), \tag{3.4}$$

where $\mathcal{N}_i(t)$ is the time-dependent set of neighbors interacting with (i), with cardinal number $|\mathcal{N}_i(t)|$, and w_{ij} is the binary weight of the $i - j$ communication link. Note that in some models, w_{ij} can take a more complicated form than our binary choice [27, 35, 36]. Using the k-nearest neighbor rule for the topological interaction distance, we have $|\mathcal{N}_i(t)| = k$ and the following dynamical equation for each individual agent in the swarm:

$$\dot{\theta}_i = \frac{1}{k}\left[(\theta_j - \theta_i) + \cdots + (\theta_{j+k-1} - \theta_i)\right], \tag{3.5}$$

where $\theta_j, \ldots, \theta_{j+k-1}$ are its k-nearest neighbors' velocity directions.

Up to this point, our modeling framework is based on a continuous-time approach. From a practical standpoint, it is necessary switching to a discrete-time approach; the associated sampling time, Δt, being intimately connected to some of the characteristic physical times of our complex dynamical system: e.g. agent's speed, speed of interagent information exchange, speed of internal information processing within one agent, etc. Once a sampling time Δt has been selected or is imposed by the natural or artificial characteristics of the system, the set of equations governing the discrete-time dynamics of the agents' property reads

$$\theta_i(t+\Delta t) = \theta_i(t) + \frac{\Delta t}{k}\left[(\theta_j(t) - \theta_i(t)) + \cdots + (\theta_{j+k-1}(t) - \theta_i(t))\right]. \quad (3.6)$$

As already mentioned, the formalism of Eq. (3.6) does not capture the fact that the value of the k indices—from j to $j+k-1$ above—are actually i-dependent since they are defined by the belonging, or not, of an agent to the neighborhood of agent i. Moreover, these k indices may change over time due to the dynamical nature of the neighborhood, which is profoundly intertwined with the dynamics of agent i (see Sect. 4.3.1).

The model devised here would not be realistic without accounting for the ubiquitous presence of noise which may have disruptive behavioral effects. This so-called behavioral noise can be divided into two broad categories: the stimulus noise and the response noise [21]. The stimulus noise, a.k.a. intensity noise, may have different origins like channel noise, environmental or background noise, and receptor noise. In the present framework, the channel, environmental and receptor noises are indistinguishable. In order to account for the global effects of stimulus noise together with external perturbing factors, a fixed level of background noise is considered throughout the agents' surroundings. In addition, the response noise may have different origins like motor noise and developmental noise which cannot be appropriately included within the present idealized modeling framework. In what follows, the response noise is therefore discarded and the stimulus noise may simply be referred to as noise without any possible confusion.

Noise can generally be assumed to be random fluctuations with a Gaussian distribution [21]. In the sequel, the background noise is considered to have a normal distribution fully characterized by its noise level, η. Specifically, the presence of noise modifies the equation governing the dynamics of agent i which now reads

$$\theta_i(t+\Delta t) = \theta_i(t) + \frac{\Delta t}{k}\left[(\theta_j(t) - \theta_i(t)) + \cdots + (\theta_{j+k-1}(t) - \theta_i(t))\right] + \eta\xi_i(t), \quad (3.7)$$

where $\eta\xi_i(t)$ is a Gaussian white noise of magnitude η since $\xi_i(t) \in [-\pi, \pi]$.

3.3 What Statistical Physics Teaches Us

Swarming systems consist of many interacting agents. There is probably no upper limit in the number of agents that can swarm together—Radakov estimated herring

schools in the North Atlantic can occupy up to 4.8 km^3 with fish densities between 0.5 and 1.0 fish/m^3, totaling about three billion fish in a single school [37]. Interestingly, the study of such highly populated swarms is greatly simplified thanks to some powerful methods of analysis borrowed from statistical physics. In the 1970s, this subfield of physics has experienced a breakthrough in the form of the renormalization group theory. Essentially, this theory says that the main features of transitions in equilibrium systems are insensitive to the microscopic details of the interactions between the parts of a system. Therefore, applying this theory to large swarming systems should appear logical as well as promising.

3.3.1 Phase Transitions

Swarming is akin to some form of coherent motion of a collective. Such coherent motion manifests itself through the existence of long-range correlations in space and/or time. These correlations can be correlations in one of the state variables, ψ_i such as the direction of travel, θ_i, as in the case where long-range order is apparent. They can also be correlations in the fluctuations, $\theta_i' = \theta_i - \langle\theta_i\rangle$, of the direction of travel in the far less conspicuous cases where apparent order is lacking [38], see Sect. 3.3.3.

Intuitively, it is clear that the ubiquitous presence of external perturbations hinders this process of self-organization. This therefore suggests the existence of a frontier separating both phases, and the crossing of this frontier is associated with a change of system-level dynamics, i.e., a *phase transition*. One of the reasons why phase transitions are studied is because they can reveal important properties of the system undergoing them. That is exactly why physicists have been engaged in such intense investigations of phase transitions in collective behaviors. Phase transitions have received much attention in statistical physics and thermodynamics where they are often tied to variations in temperature—viewed as the external "control" parameter. Examples of such phase transitions are all around us, e.g. when frozen water melts into liquid water, or when liquid water vaporizes. These phase transitions are said to be of the first order since they involve a discontinuity of one or more properties of the phase at the critical state, like density in the examples given before. Conversely, second-order phase transitions involve continuous transitions from an ordered to a disordered phase. Some variables characterizing the level of organization of the system, called order parameters, continuously decreases when the control parameter is changed. A classical example is the ferromagnetic phase transition in materials such as iron, where the magnetization—the order parameter—decreases continuously toward zero as the temperature—the control parameter—is increased above the Curie temperature.

Now turning to the case of collective motion, the most natural order parameter, and also the most commonly used, is the alignment, a.k.a. polarization, of the swarm defined as

$$\varphi = \frac{1}{N}\sum_{j=1}^{N}\frac{v_j(t)}{v_0} = \frac{1}{N}\sum_{j=1}^{N}\exp\left(\mathrm{i}\theta_j(t)\right), \tag{3.8}$$

with v_j the velocity of agent j in complex notation following the notations introduced in Sect. 3.2. This measure of the swarm order approaches the unity if all agents move more or less in the same direction and is exactly equal to the unity if they are perfectly aligned. On the contrary, if the agents fail to self-organize, $\langle\varphi\rangle \to 0$ representing utter disorder. The seminal work by Vicsek and his collaborators [6, 39] uncovered the existence of a continuous second-order phase transition with respect to two control parameters, the noise level η and the density of agents ρ. The variations of the average alignment $\langle\varphi\rangle$ with increasing noise levels η, for different densities ρ, and for both metric and topological interaction distances are shown in Fig. 3.4a, b respectively.

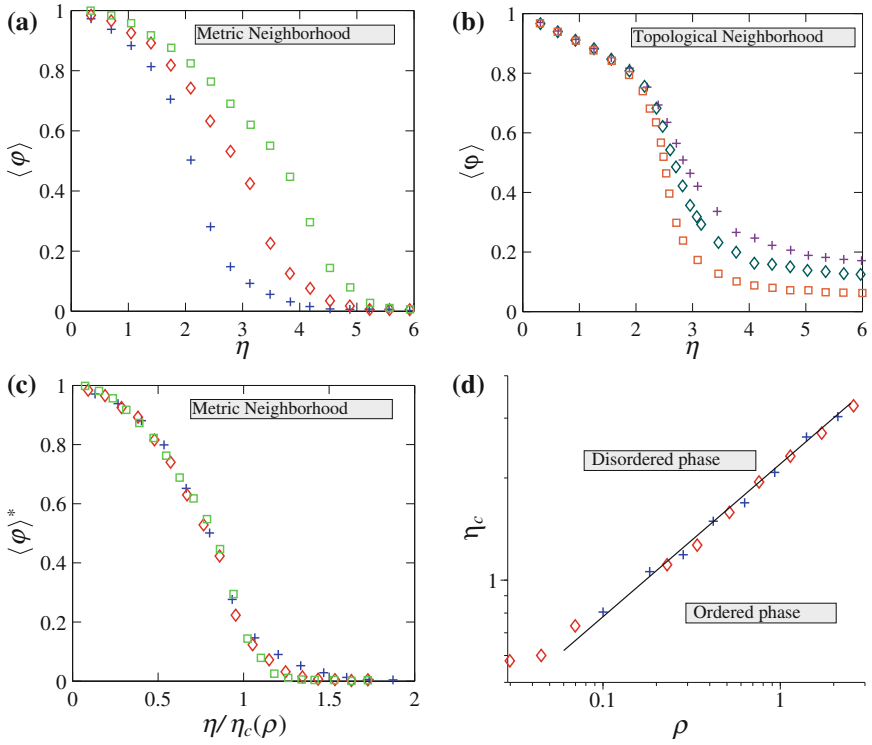


Fig. 3.4 **a** Average alignment $\langle\varphi\rangle$ of the swarm in the steady state, with a metric interaction, varying noise η for $L = 100$, and three different densities [(□) $\rho = 4$, (◇) $\rho = 2$, and (+) $\rho = 0.4$]. Data from Ref. [39]. **b** Average alignment $\langle\varphi\rangle$ of the swarm in the steady state with a topological interaction ($k = 7$), varying noise η, fixed density $\rho = 0.1$, and three different population sizes [(□) $N = 32,768$, (◇) $N = 8,192$, and (+) $N = 4{,}096$]. Data from Ref. [40]. **c** Rescaling of the phase transitions with $\langle\varphi\rangle^* \equiv \langle\varphi\rangle^*(\eta^*)$ and $\eta^* \equiv \eta/\eta_c(\rho)$ showing the universal behavior of the phase transition for all densities. Data from Ref. [39]. **d** The critical line in the (η, ρ) space of control parameters reveals a power law signature: $\eta_c(\rho) \sim \rho^{\kappa}$ with $\kappa \simeq 0.45$ (*solid line*). Data and power law from Ref. [39] for a system with $L = 100$

These results show that in the presence of weak noise $\eta \sim 0$, the swarm achieves an ordered motion, which vanishes continuously as η increases. The constant density case allows one to determine the critical value η_c of the density by looking for the divergence of the susceptibility defined as

$$\chi \equiv L^2(\langle\varphi^2\rangle - \langle\varphi\rangle^2), \tag{3.9}$$

in the thermodynamic limit as N and L go to infinity. In practice, computing extremely large SPP systems becomes quickly overwhelming. A convenient alternative consists in looking for invariance of the Binder cumulant

$$U \equiv 1 - \langle\varphi^4\rangle/3\langle\varphi^2\rangle^2, \tag{3.10}$$

with the size of the swarm.

3.3.2 Scaling and Universality

The SPP model exemplifies the emergence of long-range correlations and fluctuations when the swarm transitions from an ordered to a disordered phase. Despite the local nature of the interactions (metric or topological), structures of various scales emerge.

These features bear surprising similarities with those observed in the Ising model, which is one of the most celebrated model for the study of ferromagnetic phase transitions. In that framework, near the critical point, the system exhibits some sort of universal character associated with scale invariance properties that are characterized by a set of so-called critical exponents. For instance, the magnetization, M, in the Ising model is found to follow

$$M \sim (T_c - T)^{\beta}. \tag{3.11}$$

Similarly, the correlation length, ξ, scales as

$$\xi \sim |T - T_c|^{-\nu}. \tag{3.12}$$

In both of these scaling laws, T_c is the Curie temperature corresponding to the critical value of the control parameter. The universality follows from the fact that it is observed that these exponents are essentially independent on the microscopic details of both the interaction and the units. Each set of values of the critical exponents defines what is called a universality class.

Based on these details about the Ising model, we returned to our analysis of collective motion near the critical point. We note that the value of the critical noise depends on the density, $\eta_c = \eta_c(\rho)$, and we are in the presence of a critical line—in the space of control parameters (η, ρ)—separating the ordered phase (low η, high ρ) and the disordered one (high η, low ρ) as shown in Fig. 3.4d. Czirók et al. have shown

that in the ordered phase, and for very large systems (read $L \to \infty$) the alignment scales according to

$$\varphi \sim (1 - \eta^*)^{\beta}, \qquad \text{with} \quad \eta^* = \frac{\eta}{\eta_c(\rho)}, \tag{3.13}$$

the value of the power β found to be close to 0.42 numerically, see Fig. 3.4c. When considering the density as control parameter, we are in the presence of the following scaling law

$$\varphi \sim (\rho - \rho_c(\eta))^{\delta}. \tag{3.14}$$

Note that the classical Landau theory predicts $\beta = 1/2$, $\gamma = 1$, and $\delta = 3$ in two dimensions. However, these values are obtained within the limits of a mean-field approximation which is much simpler than the varying local interactions arising with SPPs.

Recently, a thorough comparison between the metric model and the topological one—based on an interaction with the seven nearest neighbors—has been carried out by Barberis and Albano [40]. Remarkably, the authors showed through extensive simulations that SPP second-order phase transitions have approximately the same set of scaling exponents. They also found that final cluster size distributions of the swarm have a similar power-law behavior in the metric and topological cases. These results strongly support the idea of a unique universality class, again independent of the microscopic details of the interaction.

However, these phase transitions in collective motion have been shown to actually be much more intricate than suggested by the above theory. First, it has been shown by Chaté and his co-authors that the observation of continuous phase transitions is only apparent owing to strong small-size effects [41, 42]. The thermodynamic limit has to be invoked to fully characterize the very nature of a phase transition from the statistical physics standpoint. The second source of intricacy comes from the fact that SPP systems are nonequilibrium systems unlike systems in the Ising modeling framework. This latter issue will be discussed in Sect. 3.3.4. Lastly, the way the noise η is introduced into the system's dynamics (see Eq. (3.7) for our way of doing it) has been shown to influence the observed phase transition. This technical point is thoroughly reviewed in Ref. [9].

3.3.3 Fluctuations, Correlations, Susceptibility, and Nonapparent Collective Behavior

Up to this point, we have put emphasis on the conditions required for swarms to achieve global ordering by means of local interactions, and also how the ordered phase vanishes in the presence of high noise and/or low density. As is well known from statistical physics, long-range ordering is usually accompanied by fluctuations of the state variables having very specific properties. Indeed, fluctuations in the

state variables are not random. On the contrary, these fluctuations are the inherent collective response of the swarm to external random perturbations, i.e., the noise. We will see that indeed, these fluctuations contain crucial information required for a deeper understanding of swarm dynamics.

Counterintuitively, noise and fluctuations play a key role in self-organization and in the emergence of large-scale order (see Sect. 6.3.2). The noise—often simply considered to be a white Gaussian noise—can be seen as an input signal to the open interacting multiparticle dynamical system that the swarm is. Some of the frequencies contained in the spectrum of the noise will elicit a specific response of the coupled dynamical system, in the form of fluctuations. When the system self-organizes, these fluctuations exhibit specific scaling laws that are a clear signature of swarming. It is worth adding, that recently swarming behaviors lacking global order has been uncovered for swarms of midges thanks to a careful study of these fluctuations and the associated correlations [38].

In the SPP framework, we will either consider fluctuations in the direction of travel, $\theta_i' = \theta_i - \langle\theta_i\rangle$, or more generally in the agent's velocity, $\mathbf{v}_i' = \mathbf{v}_i - \langle\mathbf{v}_i\rangle$ if we account for variations in the agents' speed. One way to analyze these fluctuations at the agent's level consists in studying how they globally affect the fluctuations of the order parameter φ. That is essentially achieved by means of a higher-order statistical analysis involving the computation of high statistical moments, such as the already mentioned susceptibility $\chi \equiv L^2(\langle\varphi^2\rangle - \langle\varphi\rangle^2)$ and Binder cumulant $U \equiv 1 - \langle\varphi^4\rangle/3\langle\varphi^2\rangle^2$. For instance, the susceptibility χ is directly related to the variance

$$\sigma^2 \equiv \langle\varphi^2\rangle - \langle\varphi\rangle^2 = \chi/L^2, \tag{3.15}$$

of the polarization φ, and near the critical point, it scales according to

$$\chi \sim (\eta - \eta_c(\rho))^{-\gamma}, \tag{3.16}$$

where γ is the critical exponent associated with the susceptibility. Note that in the thermodynamic limit, i.e., for infinitely large systems, χ diverges at the critical point.

Another classical way to analyze fluctuations lies in the study of how these fluctuations are correlated. The correlation function quantifies how local fluctuations co-vary with one another on average across space and time, thereby offering a very useful way of identifying local clusters of units behaving in a fairly similar way (see Fig. 3.5). Of utmost importance to swarm dynamics is the fact that correlations are directly related to the central process of collective indirect information exchanges mediated by local interactions. As already mentioned in Sect. 2.4.4, the ability of a system to self-organize and to collectively respond to swift changes in its environment, critically depends on the effectiveness of the social transmission of information. To quantify the size of the local clusters of coherent motion, Cavagna et al. [43] have introduced the following correlation function of the velocity fluctuations

$$C(r) = \frac{1}{c_0} \frac{\sum_{i \neq j} (\mathbf{v}'_i \cdot \mathbf{v}'_j) \delta(r - r_{ij})}{\sum_{i \neq j} \delta(r - r_{ij})}, \tag{3.17}$$

where $\delta(r - r_{ij})$ is a smoothed Dirac distribution selecting pairs of agents at mutual distance r and c_0 is a normalization factor meant to ensure $C(r = 0) = 1$.

By introducing the above correlation function and studying its variations with distance, Cavagna and his team revealed the existence of scale-free correlations in starling flocks. Such behavioral correlations of the fluctuations of orientation and speed are depicted in Fig. 3.5a for a given flock studied in Ref. [43]. When $C(r) \sim 1$, fluctuations are heavily correlated revealing a group of agents with similar behavioral dynamics. On the contrary, $C(r) \sim -1$ corresponds to antiparallel behaviors, that is, two groups tending to move in opposite directions. Furthermore, $C(r) \sim 0$ indicates decorrelation between agents and provides a quantitative measurement of the correlation length ξ, which from Fig. 3.5a is estimated to approximately 11.5 m for both speed and orientation. Interestingly, in swarming systems, the correlation length is much larger than the interaction range, which provides the swarm with a perceptual range significantly larger than its constituents'. The significance of the work by Cavagna and his team comes from the fact that by studying multiple flocks of various sizes, they found the associated correlations to be scale free. In other words, the correlation length ξ scales as a power law of the size of the flock.

Finally, note that the above two ways of studying fluctuations are related since the susceptibility is related to the integral of $C(r)$ up to its first zero, that is, up to the correlation length scale ξ.

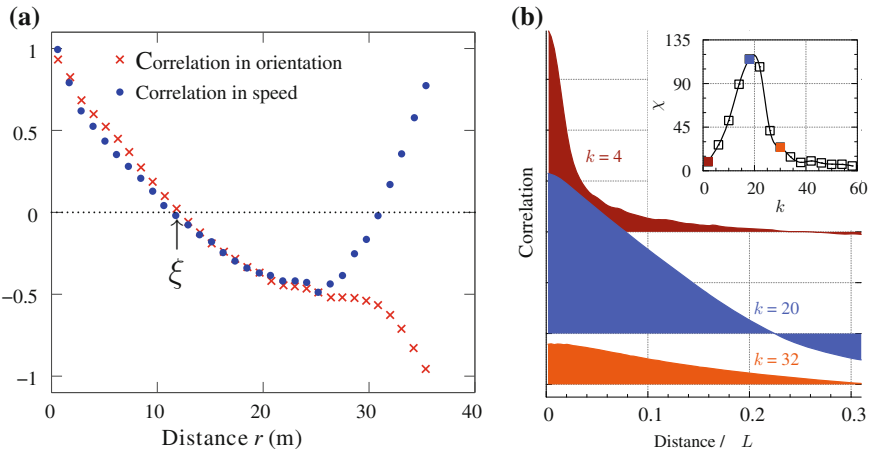


Fig. 3.5 **a** Correlations of the fluctuations of the orientation, θ'_i, and fluctuations of the speed, as a function of the mutual distance r, for a given flock based on the data from Ref. [43]. ξ is the correlation length. **b** Correlation of a SPP system ($N = 2{,}048$) based on topological interactions with different number of topological neighbors k. Results from Mateo et al. [44]. The distance is measured in units of the size of the computational domain L. The associated susceptibility χ against k is shown in *inset*

3.3.4 Nonequilibrium Systems and Self-Organized Criticality

In the previous sections, we extensively used elements borrowed from the statistical physics of equilibrium systems. However, strictly speaking swarms and SPP systems are nonequilibrium systems that are in some cases very far from equilibrium. On the one hand, the formalism from phase transitions in equilibrium systems is really elegant and helpful, and also fits very well with some of the observed collective behaviors. Nonequilibrium systems are in many cases also characterized by critical-like scaling behaviors in their fluctuations. On the other hand, there are a number of behaviors and features that simply cannot be explained by the theory of equilibrium systems.

Nonequilibrium systems are not always determined by external constraints, such as the noise level or the density in the present case. For instance, self-organized criticality may occur and does not require the tuning of any control parameter (see sandpile dynamics in Sect. 3.1.2). The transition to a self-organized critical state may simply happen by itself. Moreover, in some particular instances, nonequilibrium systems involve absorbing states which are such that the system completely looses the ability to leave this state. Typically, the unusual transition to jamming observed in some swarming systems spatially confined is a good example of such absorbing state. It was observed experimentally by Kudrolli et al. [10] with self-propelled anisotropic vibrated rods (see Sect. 3.1.3). Finally, another peculiarity of nonequilibrium systems is the appearance of very specific density fluctuations, the so-called GNF. Unlike fluctuations in equilibrium systems that scale with $\sqrt{N}$, GNF scale linearly with the number of agents N and also relax anomalously slowly [13].

3.4 What the Theory of Dynamical Systems Teaches Us

In the previous section, we presented how swarms made up of a very large number of agents could be analyzed using powerful tools of statistical physics. However, in the natural world, it is not uncommon to observe flocks of birds constituted of ten birds or less. Such 'small' flocks still are capable of amazing aerial displays. It is argued that with as few as three interacting agents, a system can develop complex adaptive behaviors. As Johnson puts it: "Two is company, three is complexity" [45]. As reviewed in Sect. 3.1.3, several artificial robotic swarms have been devised with swarm populations numbering in the tens [17] to one thousand for the kilobot experiment [18]. For these swarming systems composed of a relatively small number ($N \sim \mathcal{O}(10^1\text{–}10^2)$) of agents, a statistical analysis is completely out of question. For the sake of the discussion, let us consider $N = 10$ SPPs such that each mobile agent is governed by Eq. (3.1). This swarm is therefore a nonautonomous dynamical system with a strong dynamic coupling of the interacting components, i.e., the agents. Indeed, the modern theory of dynamical systems is focused on the investigation of systems that exhibit complex changing behaviors at the macroscopic level, emerging from

the collective actions of many interacting components [1]. In this section, we will discuss some of the central concepts of dynamical systems theory in the particular framework of swarming systems.

3.4.1 Bifurcation, Catastrophe, Collapse, and Tipping Point

Any dynamical system is either conservative or dissipative. Conservative systems are those commonly encountered in classical mechanics, and as their name suggests, they are associated with conservation properties: e.g. conservation of the total energy, conservation of linear, and angular momentum. Conservative systems are reversible and their study relies on finding a Hamiltonian, which allows to represent their dynamics in the phase space. As originally introduced by the founding father of dynamical systems theory, Henri Poincaré, the system's dynamics can be conveniently studied in a space with no explicit reference to time. The phase space—(position, velocity) or (position, linear momentum)—was introduced by Poincaré, and it is easy to show that for conservative systems, the evolution of the representative point is given by a one-dimensional curve known as the phase space trajectory. A conservative system at equilibrium has a degenerate phase space trajectory: a fixed point. An undamped oscillator admits a closed curve as phase space trajectory, and its area is conserved in connection with the conservation properties of the system. On the other hand, dissipative systems give rise to irreversible processes. Both the emergence of Bénard cells and the BZ reaction (see Sect. 3.1.2) are examples of dissipative systems. Like most natural systems, swarming systems are dissipative since they always involve dissipation at a given level. Their sustained dynamics is guaranteed by the constant influx of energy, which makes them open nonequilibrium systems.

Decades of research on dissipative systems have led physicists to realize that nonlinear effects combined with nonequilibrium constraints are at the origin of the emergence (or disappearance) of multiple solutions: i.e., a bifurcation. More specifically, a bifurcation is a phenomenon corresponding to qualitative changes in the system's dynamics following the continuous tuning of a given parameter. In the case of the Bénard cells, the parameter tuned is the temperature difference.

To better understand the concept of bifurcation and its relevance to the study of small swarms, we introduce the following one-dimensional variant of the SPP model due to Mikhailov and Zanette [46]. In that model, $N = 100$ SPPs evolve near a transition between disordered oscillating motion and coherent translational motion. The particles interact via an isotropic attractive binary potential in the presence of white noise. The governing equation for each agent (i), which replaces the discrete-time Eq. (3.7), is

$$\ddot{x}_i + (\dot{x}_i^2 - 1)\dot{x}_i + \frac{a}{N}\sum_{j=1}^{N}\left(x_i - x_j\right) = \eta\xi_i(t). \tag{3.18}$$

These dynamical equations proposed by Mikhailov and Zanette [46] for the dynamics of the swarm are second order in time and nonlinear owing to the presence of the term $(\dot{x}_i^2 - 1)\dot{x}_i$ for the velocity $\dot{x}_i$. The coupling term is an all-to-all coupling since the sum involves all the other agents—hence the long-range nature of the interaction. It is also an interaction of the consensus type based on relative states similar to the one in Eq. (3.7). The coefficient a characterizes the intensity of the global coupling term as compared to the nonlinear term.

With these nonlocal interactions and nonlinearities, the swarm becomes a multistable system that can be found in three different states depending on the initial conditions: (1) coherent traveling state to the right, (2) to the left, and (3) noisy oscillations about a certain position determined by the initial conditions. These three possible outcomes are shown in the phase portrait in Fig. 3.6a and are reminiscent of the symmetry breaking process associated with a supercritical pitchfork bifurcation. To better understand the effects of the nonlinear term $(\dot{x}_i^2 - 1)\dot{x}_i$ for the velocity $\dot{x}_i$, let us consider a single-independent particle whose velocity $u = \dot{x}$ is governed by the following nonlinear temporal evolution

$$\dot{u} = f(u) = \alpha u - \beta u^3, \tag{3.19}$$

where $\beta > 0$ and α are real coefficients. The set of fixed points (in the phase space) is obtained by having the time derivative of u to vanish, which gives

$$\lambda u^* = u^{*3} \quad \text{with} \quad \lambda = \frac{\alpha}{\beta}. \tag{3.20}$$

This simple algebraic equation admits three solutions corresponding to three fixed points. One trivial solution is $u^* = 0$, which exists under any condition. If $\alpha > 0$ and therefore $\lambda > 0$, one also obtains two nontrivial solutions: $u^* = \pm\sqrt{\lambda}$. When $\lambda \to 0$, both of these solutions coalesce into the trivial solution $u^* = 0$, hence the bifurcation. The stability of these fixed points is by the sign of

$$\gamma \equiv \left[\frac{\partial f}{\partial u}\right]_{u^*} = \alpha - 3\beta u^{*2}. \tag{3.21}$$

For the trivial solution $u^* = 0$, we have $\gamma = \alpha$ and it is therefore stable only for $\alpha < 0$. As for the two nontrivial solutions, $\gamma = -2\alpha < 0$ since their existence imposes $\alpha > 0$. We thus find them both to be stable. All the possible stable fixed points for varying λ are shown in Fig. 3.6b which explains why this bifurcation is called a pitchfork bifurcation. As can clearly be seen in Fig. 3.6b, the striking part of this bifurcation diagram is that through a continuous tuning of the parameter λ, a singularity arises in the vicinity of the bifurcation point $\lambda = 0$. This singularity yields a nonanalytic relationship between the solutions u^* and λ.

Using this simple model, we are able to better understand the emergence of very different solutions in the dynamics of the swarm model governed by Eq. (3.18) with different initial conditions. For instance, the two coherent motions in opposite

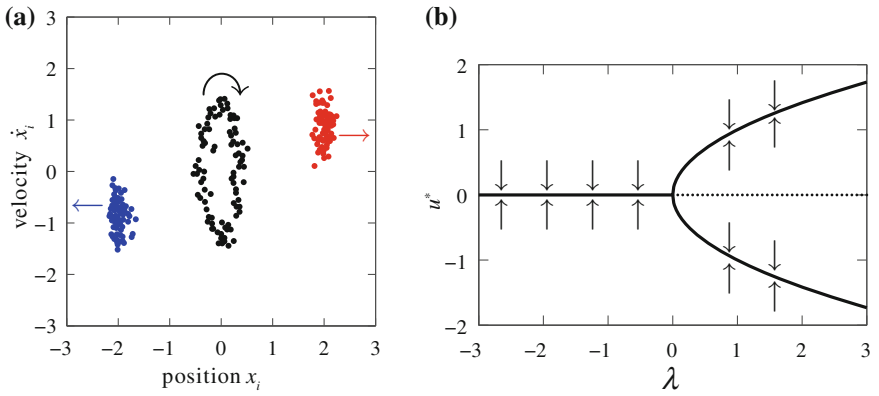


Fig. 3.6 **a** Phase portrait corresponding to three snapshots of a swarm with $N = 100$ SPPs whose dynamics is dictated by Eq. (3.18), with $a = 10$ and $\eta = 0.05$, in different dynamical regimes obtained with different initial conditions. The central ensemble (*black dots*) corresponds to disordered oscillations along a noisy limit cycle. The other two ensembles are coherently traveling ensembles toward the *right* (*red dots*) or *left* (*blue dots*). **b** Bifurcation diagram showing the stable fixed points in the velocity for the dynamical system governed by Eq. (3.19) as a function of the parameter $\lambda = \alpha/\beta$

directions correspond to two solutions on different branches of the pitchfork, past the bifurcation point. The third solution, namely the noisy limit cycle, occurs before the bifurcation point. As already described, a slow or moderate change in the control parameter (λ in the previous example) leads to abrupt shifts in the agents' state. Through the coupling of the system by means of interactions among agents, a collective shift may ensue. This process is known as percolation [47] and is often dubbed using various dramatic descriptors such as catastrophic event, collapse, or tipping point. It is worth highlighting that such collapses are also commonly encountered in other complex systems such as ecosystems—e.g. desertification of the Sahara—and social systems—e.g. propagation of fads, pandemic, to name a few [48]. The process of percolation in networked systems will be revisited in Chap. 4, while other examples of collapses will be introduced in Chap. 5.

The connection with phase transitions is now apparent. It is possible to say that phase transitions in swarming systems are akin to a collective bifurcation from the dynamical systems standpoint. Finally, it is worth adding that the study of swarms with tools and methods from the dynamical systems theory naturally integrate the important fact that these systems are nonequilibrium systems. This was not the case with the statistical physics approach that is largely based on the equilibrium systems assumption.

3.4.2 At the Edge of Chaos

A central element in the dynamical systems theory is the search for instabilities that in some cases pave the way to chaotic behaviors. The defining idea of chaos is

the possible extreme sensitivity of a system on initial conditions. For chaotic systems, minute differences in the initial state result in extremely different dynamics (or behavior) in the long run: the so-called "butterfly effect." Contrary to the colloquial sense of the word chaos, which suggests absolute randomness and the lack of order, chaotic systems exhibit substantial order. This order may not always be apparent when the system is simply observed with the naked eye but it can be detected by the physicist in the form of period-doubling phenomena and fractal-like patterns.

It is often suggested that complex systems often operate optimally in this self-organized critical state and the dynamics is said to be "at the edge of chaos" [49]. This statement, although a little misleading, means that complex systems in general, and swarms in particular, possess an adaptive degree of ordering ranging from complete order to total chaos. As was mentioned in the previous paragraph, a different kind of order exists even in the chaotic state. At this point, the existence of such a dynamic order in swarms should not come as a surprise given the emergence of scaling laws in the (nonrandom) fluctuations arising close to criticality. After all, swarms are complex adaptive systems.

3.5 Inspiration and Swarm Design

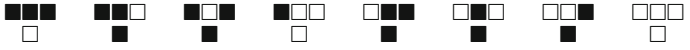
To complete this long chapter, we propose to list the central points dealt with but from the perspective of the designer of a swarming system:

- Very simple nonliving agents are capable of developing very complex adaptive behaviors. Specifically, a system does not require complicated subunits or complicated interaction rules between them. Having said that, it is worth stressing that processes of self-organization in biological systems are in general richer as compared to the same processes in physical systems, which involve nonliving components. This richness originates from the greater complexity of the living subunits, which allow for greater complexity in the interaction rules, as compared to nonliving agents solely subjected to fundamental physical laws. As stressed in Chap. 2, animals are capable of processing a lot of sensory inputs, modulate their behavior accordingly, and eventually make decisions on the basis of a high level of information. However important these factors are, they are insufficient to explain the richness and complexity associated with swarming. This richness in self-organizing dynamics can still be explored and mastered to achieve novel artificial swarming designs.
- A large number of subunits is not always necessary for swarm intelligence to kick in. Complex adaptive behaviors are possible, even with relatively small dynamical systems. However, one needs to remain fully aware of the possibility of a catastrophic event, especially transitions to absorbing states such as jamming.
- The design of the interaction rule plays a key role in the development of adaptive behaviors. This fact is well exemplified by cellular automata. We saw in Sect. 3.1.1 and in Fig. 3.1, how rule 110 yields a very rich behavior with a certain level of

dynamic order with fractal-like patterns appearing. Comparatively, rule 108 given below

Rule 108 

and which is extremely close to rule 110

Rule 110 

produces an absorbing state in the form of a fixed point lacking any adaptivity.

- If one design can afford a very large population of swarming agents, the microscopic details of the interaction become less critical. Yet, to increase the agents' perceptual range so as to allow a coordinated response to emerge, the swarm has to be operate close to criticality. Unfortunately, the actual self-positioning close to criticality is probably a very stiff problem. Indeed, the theory of self-organized criticality tells us that nonequilibrium systems do not required the tuning of any control parameter. The transition is spontaneous.
- Collective actions can be designed at a priori any spatial scale. It is therefore not unrealistic to envisage swarm of microbots collectively and autonomously operating inside the human body with the aim to achieve minimally invasive forms of medical treatment.
- Swarms are to be designed to promote adaptivity and as we have seen in this chapter, adaptivity is highly dependent on dynamic ordering, i.e., to be at the edge of chaos.
- The environmental noise and other sources of perturbations help generate fluctuations at the swarm level that have a very specific signature. These fluctuations contribute to the emergence of an adaptive behavior. Unfortunately, from the design standpoint, this fact cannot utilized as in general noise is not a controllable parameter.

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Chapter 4
A Network-Theoretic Approach to Collective Dynamics

Characterizing the dynamics of a swarm as we did in the previous two chapters is only one step toward understanding it. We also need to understand how local interactions influence the overall system's dynamics. The modern science of networks provides a very elegant and powerful framework—essentially grounded in graph theory—to bridge the gap between local dynamics and interactions at the agents level and global response at the swarm level. Indeed, network models offer a natural way of describing how self-organization arises in complex systems, which in turn helps us gain insight into dynamical processes occurring on them. Moreover, network science has a lot in common with statistical physics: percolation, scaling, order parameters, renormalization, self-similarity, phase transitions, and critical exponents were introduced in Chap. 3 in the context of swarm dynamics, and they remain highly relevant for a network analysis [1]. As was noticed with our analysis of correlations of fluctuations in Chap. 3, it can be really challenging to identify emerging patterns and their properties, especially for swarms lacking apparent order. Network theory provides yet another invaluable toolbox to uncover "hidden" structures emerging through self-organization.

4.1 A Science of Networks

Over the past decade, network science has emerged as a new interdisciplinary field allowing scientists and engineers to represent and analyze the intricate underlying structure of connections among elements of complex systems. In its most basic sense, a network is any collection of units—the nodes or vertices—in which some pairs of these objects are connected by links—the edges. Clearly, such a high level of abstraction applies to any system thereby providing a conceptual representation of interrelations between interacting units. It is important emphasizing that the network approach is intentionally limited to mapping the interactions between units, with all the attention focused on the global structure of the interactions within a system. The

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detailed properties of each element or unit on its own are simply ignored and not accounted for at the network level: this is the so-called "network thinking" paradigm.

Analyzing widely different systems—i.e., connected parts—using the same representation can only be achieved using a high level of abstraction. As expected, there is an unavoidable loss in certain levels of detail in the system representation. In many instances, this loss in details conveniently yields a bare structure from which universal characters can be singled out. Why are complex networks such as hub-based airline networks and electrical grid networks prone to similar catastrophic and cascading disruptions? Also, why is the spread of a contagious disease such as the flu so similar to the propagation of a new fad in social networks? Can the amazingly fast propagation of behavioral information in a school of fish be compared to the self-reinforcing cellular signaling in aggregating amoebae? Such questions can found an answer in network science as it offers unique ways to identify commonalities between networks originating from very different domains.

Like any discipline, network science has developed its own lexicon, partially inherited from graph theory. For a complete introduction to network science and graph theory, we refer the reader to specialized monographs such as Refs. [2, 3]. However, some network-theoretic notions are required for our discussion about swarm signaling networks. We will therefore provide in the next paragraph some basic, possibly incomplete and not general, definitions of these key concepts.

The concept of distance in networks has to be understood in terms of the minimum number of hops between any two vertices. In general, there are many possible paths between any pair of nodes, and the shortest path gives a sense of such network distance. For the network as a whole, the *average* (over all pairs of node) *shortest connecting path* ($SP = \ell$), $\langle \ell \rangle$, gives an indication of the overall distance between vertices. Many networks appear to be *'small world'* as was dubbed by Watts and Strogatz [4]. This very important property can readily be understood in the frame of social networks, whereby anyone of us on the Earth is, on average, six social links—six degrees—away from virtually anyone else [5]. The concept of *connectedness* refers to the existence of a path between any pair of nodes, such that exchanges between any two agents are in theory possible. Another important metric, *the clustering*, is related to the existence of distinct tight-knit communities in networks. Specifically, for a given node, the clustering coefficient *CC* measures to what extent this node's neighbors are also neighbors of each other. It turns out that there is a clear relationship between the speed by which information is disseminated in social networks and the clustering coefficient: the higher the degree of clustering, the slower the dissemination [6]. Finally, a key structural property of networks is the *degree distribution*, which gives an indication of the number of edges coming into (or out of) each and every node. If you have, say 70 connections in your preferred online social network, then your degree is 70. High-degree nodes are often call hubs like in transportation networks; they play a vital role in redirecting information throughout the whole network and any disruption affecting it ends up creating major failures in the dynamics of the system. Many real networks possess hubs and their degree distribution is said to be *'scale free'*; on other words its degree distribution is governed by a power law as was discussed in Sect. 3.3.3 for correlations.

4.2 Swarm Signaling Networks

The network approach seems very natural when dealing with social networks, electrical power grid, road networks, etc. This has probably something to do with the fact that many visual representations of these networks are commonly available and are frequently showcased, see Fig. 4.1. Interestingly, this very same approach initially appeared far less natural to scientists—primarily physicists, ethologists, and biologists—working on gaining a better understanding of swarms and their dynamic collective behaviors. With hindsight, this could appear surprising for at least three reasons. First, a network-theoretic approach has already been successfully considered to decipher some intricate social animal behaviors [7]. Second, the network approach focuses on relationships between entities rather than entities themselves, and these very relationships were acknowledged to be at the root of self-organizing and emergent collective behaviors. Third, many emergent phenomena crucially depend on the topology of the underlying networks.

One possible reason justifying the late adoption of the network paradigm by researchers studying collective behaviors may originate from the nontrivial dynamics of the relationship between interacting individuals. Indeed, networks such as for instance social networks and the electrical power grid networks are not static networks. They can grow or decay and their topologies evolve with the possibility of hubs emerging or declining, communities or cliques forming, etc. In general, their dynamics and therefore the dynamics of their topology occur at timescales significantly smaller than typical timescales associated with dynamic processes occurring on these networks. For instance, the propagation of a new fad through a social network is in general a faster process than the one corresponding to the formation of this social network, although both processes unfold simultaneously. However, when considering dynamic collective behaviors, the swarm signaling network (SSN)—or simply signaling network, a.k.a. interaction network—embodies, at any given instant, the state of interagent signaling or interaction. As a consequence, the structure of the

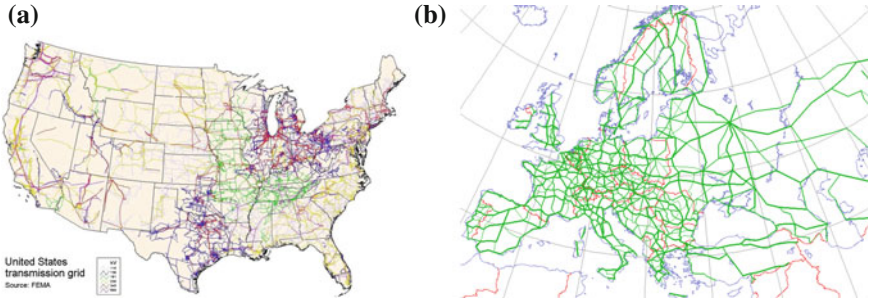

Fig. 4.1 **a** United States power transmission grid. Different colors correspond to different high voltages. **b** European E-Road network

SSN changes in response to the ongoing dynamics on the network, thereby creating a feedback loop between the dynamics of the swarming agents and the adaptive topology of interaction.

As was discussed in Chap. 3, in the presence of appropriate conditions, the collective dynamics of a system of locally interacting units may result in self-organization. We saw that such emergent behaviors come in different forms and sizes [8]. Many examples of swarms encountered in the natural world were briefly discussed in Chap. 2. Here, we would like to give the reader other examples to stress one important point: a vast spectrum of spatiotemporal dynamics are achieved—persistent versus transient ordering, localized versus global ordering—with often multiple signaling modalities required for the interagent communication. In Chap. 3, the swarming behaviors considered were solely based on physical interactions. Trophic interactions and informational exchanges (see Sect. 2.4.1) offer ways to achieve social transmission of information in more elaborated and complex manners. For example, in the ocean environment, it is not uncommon to encounter shoals of fish maintaining a certain level of ordering in the heading of swimming individuals, and that, across distances that can go beyond several kilometers [9]. Shoaling fish can shift to a more cooperative mode forming coordinated schools, then shift back to an amorphous shoal within seconds. Typically, the level of global ordering in a school of fish is relatively high in the presence of sufficient ambient light. Fish forming a school disperse at dusk as a consequence of their reduced ability to socially interact and exchange behavioral information. Flock of birds—among which the starlings are the most commonly studied—also switch between very ordered phases while traveling collectively and highly fluctuating ones after returning to their roosting sites. At a smaller scale, locusts are also well known to generate highly ordered marching behaviors beyond a certain critical density of individuals [10]. Interestingly, some other insects, such as midges, exhibit collective behavior in the absence of collective order [11]. At an even smaller scale, the amoeba *Dictyostelium discoideum* is well known for its switch from a disordered collection of independently foraging cells to a social collective aggregation triggered by the absence of food in their natural environment. This extensively studied collective aggregation of amoebae is the outcome of complex local signaling interactions of both chemical [12] and mechanical [13] origins, with additional influences imposed by the physicochemical nature of the extracellular environment [14]. In summary, using various communication channels, information may be exchanged using multiple modalities (e.g., vision and lateral sensing are both required for fish to school), in a directional (e.g., one bird take cues from its neighbors without them noticing it) or undirectional way, in a direct or undirect way (e.g., stigmergy in social insects), etc.

It therefore becomes apparent that a general analysis of dynamic collective behavior and swarming cannot be achieved with a modeling approach solely based on specifying local interaction rules or behavioral algorithms at the agent level [15]. The issues with such local-to-global analyses are even more apparent when designing artificial collective behaviors aimed at achieving a predefined task—a formidably ill-defined and wicked problem. Instead, all modeling approaches of dynamic collective

behavior should allow one to deal with the swarm as a whole—the superorganism paradigm—by means of a global representation of some sort. Readers familiar with engineering systems analysis would welcome this as being the 'natural' and 'classical' approach. This approach is by no means specific to dynamic collective behaviors; it actually pervades the systems world, where a global analysis of the architecture of systems is required to analyze function, control, stability, etc. Among all the paradigms available to carry out a holistic analysis of collectives, the *network-theoretic* one appears to be one of the most appropriate.

4.3 Network Properties and Swarm Dynamics

4.3.1 Assembling the Swarm Signaling Network

The landmark study from the STARFLAG group imaged and tracked wild flocks of starlings numbering in the thousands and revealed that the neighborhood of interaction actually depends on a topological distance [16] as was discussed in Sect. 3.2.2. More precisely, Ballerini et al. [16] discovered that each bird interacts on average with a fixed number of nearest neighbors (six to eight). Following a network approach, one can therefore explicitly assemble the SSN for starlings flocking. For definiteness, if one assumes that starlings only interact topologically with exactly k of their nearest neighbors, then the signaling network maintains a constant number of nodes and a constant number of edges. However, at each instant a certain number of link failures occurs which exactly amounts to the number of newly created ones. The rate at which those links are destroyed and created is governed by the pace of the physical dynamics of the flock as well as the degree of alignment—or long-range polarization. Hence, we are in the presence of a switching network whose switching dynamics is directly associated with the dynamics of the nodes themselves, i.e., the dynamics of starlings.

As was mentioned in the previous section, there exists a feedback loop between the dynamics of the swarming agents and the adaptive topology of interaction. Networks, including SSNs, containing such a feedback loop are referred to as temporal or adaptive networks, and are starting to receive more attention [17, 18]. Indeed, most of the traditional modeling frameworks in network science fail to accommodate the challenges induced by the adaptive nature of a network. Some of these approaches may have to be extended and new frameworks should be sought and developed in order to address the dual nature of adaptive networks. In this book, we will emphasize a specific adaptive network, namely the SSN corresponding to a topological interaction, as in the particular case of flocks of starling. However, this example is meant to illustrate the key ideas behind the general concept of swarm signaling network.

We now precisely define and construct the SSN which, as already mentioned, is the underlying information transfer channel behind the dynamics of the interacting swarming agents. Constituent links of the SSN of a group whose agents have directed

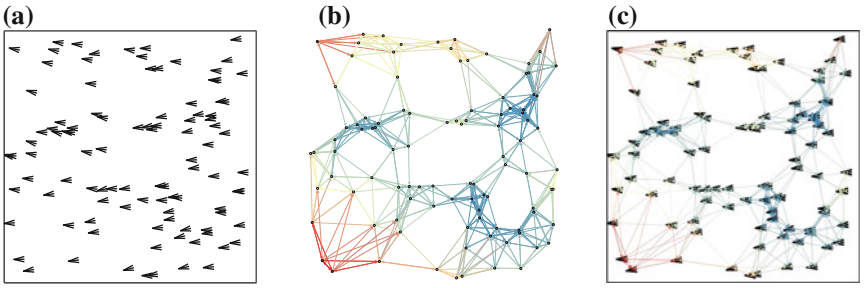

Fig. 4.2 **a** Physical view: snapshot of a swarm of $N = 100$ topologically interacting individuals traveling at constant speed in a two-dimensional square domain (10×10) with periodic boundaries; each agent interacts topologically with $k = 7$ neighbors. **b** Network view: the associated swarm signaling network (SSN); the nodes and edges are colored according to the topological distance (increasing topological distance from *blue* to *red*). **c** Combined view: the swarm overlaid with the SSN

interactions are unidirectional by opposition to bidirectional interactions in a group of agents with undirected interaction edges. The topological neighborhood of interaction (see Sect. 3.2.2) based on the k-nearest neighbor rule allows one to locally identify the links between agents. The topological character of the neighborhood of interactions has a tremendous impact on the properties of interagent connectivity, in particular with the induced asymmetry in the relationship whereby if agent j is in the neighborhood of agent i, then i is not necessarily in the neighborhood of j, i.e., the interaction is directed. On the contrary, with a metric neighborhood the interagent connectivity is fundamentally symmetric with the presence of undirected interactions.

Through a bottom-up assembly of the interagent links, the complete global graph characterizing the connectivity can be constructed as shown in Fig. 4.2. Given the dynamics of the topological neighborhood and the directed nature of the links, the SSN is a switching strongly connected k-nearest neighbor digraph [19–21]. It is worth noting that the random graph theory [22–26] is neither appropriate, nor relevant to the study of the dynamics of the connectivity in swarms since links are introduced irrespective of any distance between nodes—be that in the physical space or in the signaling network space.

Finally, it is important noting one key specificity of SSNs. By construction, their nodes are embedded in the physical space and this has far-reaching consequences from the design standpoint. Specifically, establishing a long-distance connection is virtually impossible in natural swarms owing to the natural limitations in signaling range. Therefore, the properties of the SSNs for natural swarms display a spatial bias similar to the one encountered in infrastructure networks (see Fig. 4.1). However, when designing artificial swarms, it may become possible (and probably desirable as we will see) to establish long-(spatial)-distance connections. This latter point is discussed at length in the upcoming Sect. 4.4.

4.3.2 Connectedness of the Signaling Network

Within our SPP modeling framework (see Sect. 3.2), the adaptive swarm signaling network (SSN) is explicitly accessible and one may ponder over the details of the relationship between connectedness of this network and emergent collective behaviors through local synchronization. Indeed, focusing on the network topology allows one to see that physically distant agents are actually strongly connected through very short paths. Here, we propose to bridge the gap between two vastly different representations of the dynamics of our complex adaptive system. On the one hand, we have the prevalent canonical representation in the physical space—e.g., kinematic tracking of group members—and, on the other hand, the SSN approach in the 'network space.'

In the physical space, the emergent outcome appears before one's eyes (Fig. 4.3a–c). Reaching local synchronization is a key factor in forming a group and maintaining its emergent behavior, otherwise the group will split apart unless a consensus is reached again. Furthermore, consensus decisions bring along enhancement of decision accuracy compared with lone individuals and improvement in decision speed [27, 28]. For a group to self-organize, the union of the dynamically evolving SSNs must have a spanning tree frequently enough [29]. Empirical evidences implicitly indicate the existence of a signaling channel between every two arbitrary agents in the swarm at any point in time. From the unique observations and findings of the STARFLAG project, Cavagna et al. [30] came up with this compelling statement: "The change in the behavioral state of one animal affects and is affected by that of all other animals in the group, no matter how large the group is." As discussed in Chap. 3, this fact can be understood from the statistical physics standpoint, given the observed scale-free nature of the correlations of fluctuations, see Sect. 3.3.3. However, this fact can also be fathomed from the network-theoretic standpoint, in relation with the property of connectedness of the SSN. Formally put, the SSN of the swarm is strongly connected at all time which is a much stronger condition than the one presented in Ref. [29].

The very first characterization of the SSN pertains to its connectedness, which, in a k-nearest graph representing the topological interactions (see Sect. 3.2.2), heavily depends on the value of the outdegree k (Fig. 4.3d, e). The existence of a critical value, k_c, for the outdegree k such that for $k \geq k_c$ the k-nearest graph is connected, has never been proved. However, Balister et al. [20] proved the existence of k_c in the probabilistic sense. More specifically, they proved that for

$$k \geq k_c = c \log N, \tag{4.1}$$

where N is the number of nodes—i.e., the number of swarming agents—the probability for any randomly generated k-nearest graph to be connected tends to one. In Eq. (4.1), c is a constant and the smallest value found so far is 0.9967 [20]. It is important keeping in mind that those mathematical results were obtained under the assumption that N is large. When collective motion is considered, the number

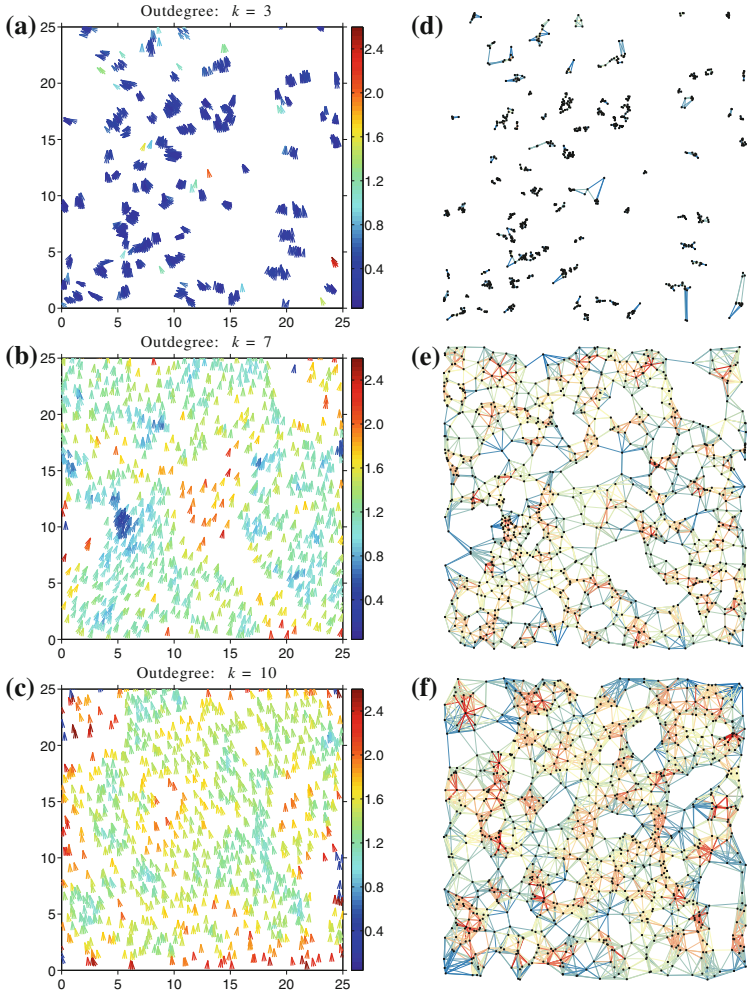

Fig. 4.3 At a given instant, in a quasi-steady-state regime, velocity directions θ_i of $N = 1{,}000$ agents are displayed in the physical space (*left column*) and the associated SSN in the network space (*right column*) for three different values of the outdegree k: **a** and **d** outdegree $k = 3$; **b** and **e** outdegree $k = 7$; **c** and **f** outdegree $k = 10$. *Left column* the actual velocity of an agent is indicated by a *small arrow* which color is mapped onto the size of the radius of the topological neighborhood of interactions. The vertical colormap is identical for all values of k, and the size of radius is expressed with the same spatial units as the square domain $[0, 25]^2$. Roughly, a *blue arrow* corresponds to an agent with a fairly small topological neighborhood of interactions, while, on the contrary, a *red arrow* indicates a large topological neighborhood of interactions. *Right column* instantaneous SSN associated with the physical distribution of agents shown in the *top row*. The network nodes are exactly located at the agents' physical locations. The directed links are colored according to the value of the indegree k_{in} of the source node, also colored, from which they are originating. A linear colormap ranging from *blue* to *red* is used with three different indegree intervals: $k_{\text{in}} \in [0, 8]$ for $k_{\text{out}} = 3$, $k_{\text{in}} \in [1, 13]$ for $k_{\text{out}} = 7$ and $k_{\text{in}} \in [3, 17]$ for $k_{\text{out}} = 10$. The results correspond to the time step $t = 3{,}000$ nondimensional time units, in a quasi-steady state. The noise level is fixed and set to $\eta = 0.1$

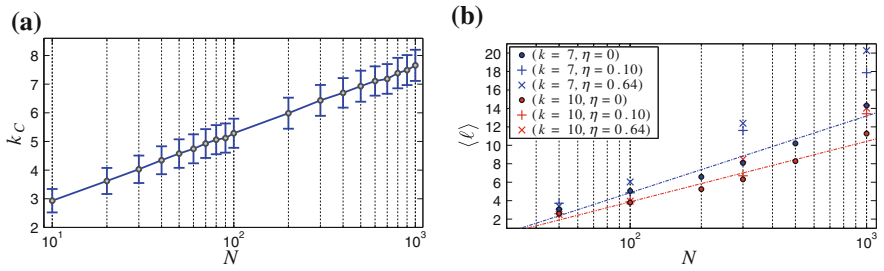

Fig. 4.4 **a** Critical value of the number of topological neighbors, k_C, for which the connectedness of the network is guaranteed, as a function of the swarm size N, with N ranging from 10 to 1,000. *Gray dots* represent the average value of k_C obtained from a statistical analysis comprising 1,000 randomly generated k-nearest digraphs. The *errorbars* represent the associated standard deviations. **b** Average shortest connecting path $\langle \ell \rangle$ versus number of agents for the SSN. A log scale is used for the number of agents N. Two possible values of the outdegree are considered: $k_{\text{out}} = k = 7$ and 10. Three values of the noise level η are considered: noiseless ($\eta = 0$), moderate ($\eta = 0.1$), and high ($\eta = 0.64$). The linear fitting in log scale is only shown for the noiseless case using *dash-dotted lines*

of agents considered ranges from dozens to a few thousands, and rarely more [8]. It is therefore important to assess numerically the validity of Eq. (4.1) for values of N smaller than 1,000. Figure 4.4 shows that even for small values of N, k_C continues to scale linearly with $\log N$ on average. Moreover, the average value of the coefficient c here is found equal to 1.15—this value decreases with increasing N, which is consistent with the value 0.9967 found in Ref. [20] for large N.

The study of the connectedness of the SSN uncovers the existence of a relationship between swarm size N and the number k of nearest neighbors influencing any agent's behavior. Indeed, general results from graph theory applied to the study of the SSN connectedness take a particular significance in the context of dynamic collective behavior where N may not necessarily be very large and k, cannot possibly exceed at most 15 to 20 due to the intrinsic bandwidth limitations in signaling, sensing and internal information processing. To better appreciate these results, the dependence of the probability of connectedness of the SSN as a function of N for different values of k is depicted in Fig. 4.5, which reveals the profound relationship between connectedness of the swarm and the number of agents N, for different values of the outdegree k. This important result was already suggested by Eq. (4.1) and is related to the concept of percolation at critical connectivity. As was seen in Sect. 3.4.1, at the critical point, system-level communications become possible and information can flow between agents throughout the entire swarm, and in a very effective manner. In addition, with adaptive networks, there is a feedback of global information into the topological evolution through the local agent's dynamics that can allow the SSN to self-organize toward a critical state.

For the sake of explanation, let us consider a swarm comprising $N = 1{,}000$ agents, which is a reasonable number for living animals [31]. Figure 4.5 shows that this swarm will remain connected at all time if k has at least a value of approximately

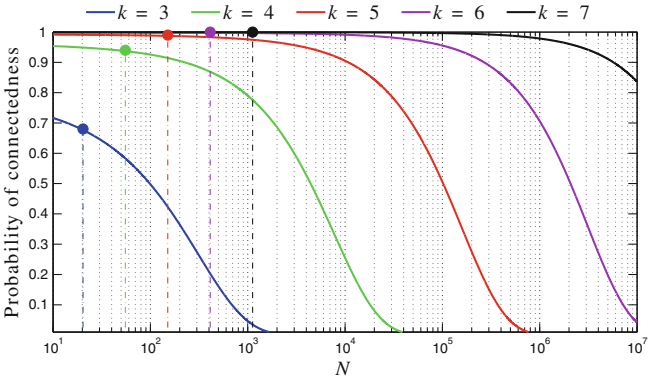

Fig. 4.5 Probability of connectedness for the SSN versus number of agents N for different values of the number of nearest neighbors k. The SSN corresponds to a specific configuration of the swarm in which N nodes are placed in a unit square independently through a uniform distribution. Then each node is connected to its k nearest neighbors to form the k-nearest graph. For each value of the outdegree k, the maximum size of the swarm population N_C—given by $k = k_C = c \log N_C$ with $c = 0.9967$ [20]—ensuring the connectedness of the SSN is represented by a *colored dot* with the associated *vertical dashed line*

6 or 7. This result is in very good agreement with the experimental observations by Ballerini et al. [16] for flocks of starlings with approximately 1,000 $\sim$ 1,200 birds at maximum. Based on their thorough analysis of the dynamics of flocks, Ballerini et al. [16] claimed that each starling interacts topologically with 7 other birds on average. Thus, this general analysis of swarms based on network-theoretic elements leads to a more general rule of interaction in swarms: each agent interacts on average with a fixed number of neighbors irrespective of the distance, and that number of neighbors k depends on the swarm size N. By extension, for artificial swarms, which typically have a much smaller size—with say N being at most 100—this analysis enables us to conclude that 4 to 5 interacting neighbors are necessary to ensure the swarm's connectedness and effectiveness. Note that, this analysis based on Fig. 4.5 does not account for the dynamics of the SSN and more importantly for the ubiquitous presence of noise in the environment.

4.3.3 Shortest Connecting Path

Let us first consider the distance among agents in the swarm, and by distance here, we mean the network distance between nodes representing the agents in the swarm network, and not the physical distance between agents in the physical space. This note is important as in the particular case of swarms, there is a nontrivial relationship between network distance and physical distance. As was introduced in Sect. 4.1, a good metric for network distance is given by the shortest connecting path, $SP = \ell$,

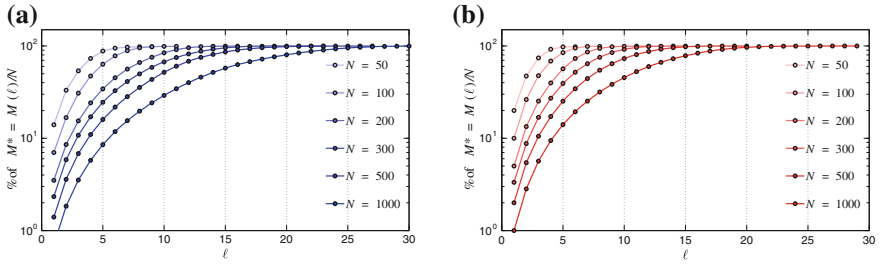

Fig. 4.6 Normalized hop plot: $M^* = M(\ell)/N$ for the SSN. A log scale is used for the number of agents M and various swarm sizes N are considered. Two possible values of the outdegree are considered: **a** $k_{\text{out}} = k = 7$; **b** $k_{\text{out}} = k = 10$. The noise level is fixed and set to $\eta = 0$

between any pair of agents. This metric is intimately related to the small-world effect [4], with which it is possible to go from one agent to any other in the swarm passing through a very small number of intermediate agents. To be more precise, the small-world property refers to networks in which the average shortest connecting path, $\langle \ell \rangle$, scales logarithmically, or more slowly, with the number of agents N. Figure 4.4b illustrates the average shortest connecting path $\langle \ell \rangle$ versus N for two different outdegree values $k_{\text{out}} = k = 7$ and 10 for our SSN, and for three vastly different noise levels—noiseless, moderate, and high. Those values for k were chosen so as to ensure that the network remains connected for up to $N = 1{,}000$ agents—the connectivity being necessary to compute the average shortest connecting path. Given the log scale on the x-axis, it is clear that interaction networks for swarms exhibit the small-world phenomenon. This result is further supported by a very recent mathematical analysis by Alamgir and von Luxburg [32]. Not surprisingly, a higher outdegree shortens the shortest connecting path for all swarm sizes. On the contrary, $\langle \ell \rangle$ is lengthened when the swarm evolves in increasingly noisy environmental conditions, but the small-world property is conserved.

The small-world property can be more thoroughly analyzed by inspecting the behavior of the quantity $M(\ell)$ defined as the average number of agents within a network distance less than or equal to ℓ from any given agent [22]. The corresponding hop plot is shown in Fig. 4.6 for two values of the outdegree $k_{\text{out}} = 7$ and $k_{\text{out}} = 10$. The exponential increase of M with ℓ is yet another proof of the small-world character of the SSN.

4.3.4 Clustering Coefficient

It is very interesting to observe that our swarm model generates a SSN showcasing the small-world effect. However, in many social and technological networks, the small-world effect is accompanied by a relatively high level of clustering. For instance, random networks also exhibit the small-world effect but possess an extremely low level of clustering.

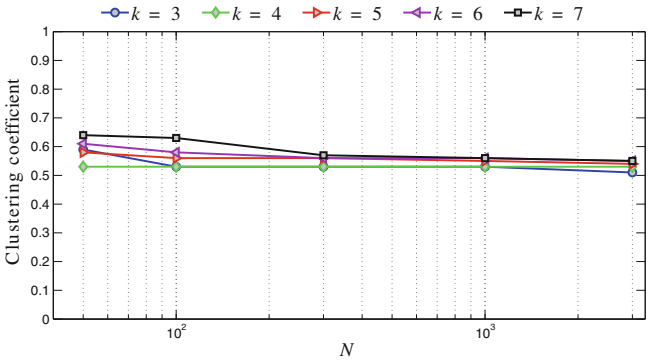

Fig. 4.7 Clustering coefficient (CC_{out}) versus number of agents for the SSN. A log scale is used for the number of agents N. Different values of the outdegree are considered: $k_{\text{out}} = k = 3, \cdots, 7$. The noise level is fixed and set to 0

As was introduced in Sect. 4.1, the clustering coefficient, CC, characterizes the local cohesiveness of networks as well as the propensity to form clusters of interconnected elements [4]. Given the directed nature of the SSN and the fact that neighbors are pointed at by outward edges, we consider the extended definition of the clustering coefficient CC_{out} given in Ref. [33]. Thus, the average clustering coefficient of our k-nearest neighbor graph can be calculated as follows [33]:

$$CC_{\text{out}} = \frac{1}{k(k-1)N} \text{trace}(A^2 A^{\text{T}}), \tag{4.2}$$

where k, N, and A are the outdegree, the number of agents, and the adjacency matrix of the SSN, respectively [2]. Figure 4.7 shows the swarm's clustering coefficient as a function of the number of agents N in the swarm, for several different values of the outdegree k, and in the absence of noise. These results highlight the rather high independence of the clustering coefficient with both the number of agents and the outdegree. We are therefore led to conclude that the SSN is intrinsically highly clustered unlike random networks. Interestingly, those measured levels of clustering are practically not affected by the presence of environmental noise—moderate ($\eta = 0.1$) and high ($\eta = 0.64$) noise levels were tested. We contend that the high level of clustering in the SSN may find its origins in the existence of clusters of agents in swarms, as commonly observed in nature [31].

4.3.5 Degree Distribution

We have seen in the previous sections that the SSN is a clustered small-world network. To better understand its subtle structural organization, we now turn to the study of its statistical homogeneity. Homogeneous networks are characterized by fast-decaying degree distributions, whereas heterogeneous networks produce long and heavy tails—such power laws are a well-known signature of scale-free networks [22].

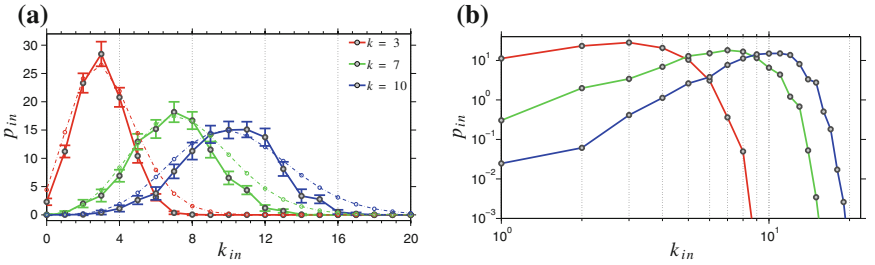

Fig. 4.8 Indegree distribution p_{in} of agents in the SSN for several swarms with $k = k_{\text{out}} = 3, 7,$ and 10 and different number of agents $N = 50$, 300, and 1,000; **a** linear scales with the exact values corresponding to the Poisson distributions for $k = 3, 7$ and 10 shown using *thin dash-dotted lines*; **b** logarithmic scales. The average indegrees $\langle k_{\text{in}} \rangle$ are 3, 7, and 10 and their standard deviations $\sigma_{k_{\text{in}}}$ are approximately 1.4, 2.2, and 2.4, for $k = k_{\text{out}} = 3, 7,$ and 10, respectively. The noise level is fixed and set to 0. The averaging $\langle \cdot \rangle$ considered is a mixed conditional averaging based on a temporal averaging of the network configurations for 800 consecutive timesteps—with $\Delta t = 1$—repeated 8 times each, and that for three different values of the total number of agents: $N = 50$, 300, and 1,000

The indegree, k_{in}, of an agent in the SSN is the number of directed edges pointing at it; a directed edge representing a neighboring agent using the information from the state of the agent that its edge is pointing at. The indegree distribution, $p_{\text{in}}(k_{\text{in}})$, is the fraction of agents in the SSN having an indegree k_{in}. The average indegree distribution, $\langle p_{\text{in}} \rangle$, for our SSN is computed for three distinct values of the outdegree, $k = 3, 7,$ and 10 as was done previously for our study of the connectedness. It is important to note that very little variation in the average indegree distributions is observed for the three values of N considered, as shown in Fig. 4.8, in which the errorbars represent the standard deviation to the average value found. The indegree distributions are peaked at $k_{\text{in}} = k_{\text{out}} = k$ for the three values of the outdegree considered. More precisely, approximately half of the swarm agents have an indegree such that $k_{\text{out}} - 1 \leq k_{\text{in}} \leq k_{\text{out}} + 1$. Furthermore, for $k = 7$ and $k = 10$, the indegree distribution is qualitatively symmetric about their maximum value obtained at $k_{\text{in}} = k_{\text{out}}$. Based on the log–log plot of the indegree distribution in Fig. 4.8b, it can be said that the indegree distributions clearly are Poissonian like, with $\langle k_{\text{in}} \rangle = k_{\text{out}}$ and with a variance increasing with $k_{\text{out}} = k$. This is further verified by comparing the results with the actual Poisson distribution as shown in Fig. 4.8a with a relatively good qualitative agreement. Such Poissonian-like distributions are reminiscent of random networks and starkly differ from power laws characteristic of scale-free networks. Similarly to the clustering coefficient, measured indegree distributions are practically not affected by the presence of environmental noise—moderate ($\eta = 0.1$) and high ($\eta = 0.64$) noise levels were tested.

The heterogeneity parameter $\kappa = \langle k_{\text{in}}^2 \rangle / \langle k_{\text{in}} \rangle$ allows one to further confirm the absence of an intrinsic characteristic scale for the SSN. Homogeneous networks are known to have a κ that scales with the indegree k_{in} [22]. Table 4.1 shows the values of the reduced heterogeneity parameter $\kappa^* = \kappa / k_{\text{in}} = \langle k_{\text{in}}^2 \rangle / \langle k_{\text{in}} \rangle^2$ for 9 SSNs corresponding to three values of the outdegree $k_{\text{out}} = 3, 7,$ and 10, and for 3 different sizes of swarms corresponding to $N = 50, 300,$ and 1,000 agents. These

Table 4.1 Reduced heterogeneity parameter $\kappa^* = \kappa/k_{\text{in}} = \langle k_{\text{in}}^2 \rangle / \langle k_{\text{in}} \rangle^2$ for 9 SSNs corresponding to 3 values of the outdegree $k_{\text{out}} = 3$, 7, and 10, and for 3 different sizes of swarms corresponding to $N = 50$, 300, and 1,000 agents

N	$k_{\text{out}} = 3$	$k_{\text{out}} = 7$	$k_{\text{out}} = 10$
50	1.21	1.12	1.10
300	1.21	1.10	1.06
1,000	1.31	1.09	1.08

results confirm the homogeneity of all SSNs as κ indeed scales with the indegree k_{in}, irrespective of the outdegree and swarm size. With that, one can conclude that SSNs are homogeneous and clustered small-world networks.

With the above structural details—shortest connecting path, clustering coefficient and indegree distribution, one can conclude that, if connected, the SSN is a homogeneous and clustered small-world network even when considering the disruptive effects of noise on the interagent interactions. Hence, the swarm information transfer channel has a relatively high-local cohesiveness, and no intrinsic characteristic scale could be found in the indegree distribution. The small-world phenomenon could have been intuited through the mere observation of exceptionally fast responses of biological swarms to external cues, e.g., fish school evasive maneuver, collision avoidance, etc. The homogeneous character of the SSN could also have been intuited. Indeed, the difference in indegree distribution has vastly significant implications for the structure of the networks. For instance, the long tail of power-law distributions of the indegree is a clear signature of the existence of hubs in scale-free networks. Interestingly, even though the swarm interaction network is not, per se, a random network—its dynamics is governed by a set of rules, including the k-nearest neighbor rule—its indegree distribution is not able to reflect those differences with real random networks. Note that, this result is not surprising given that we are dealing with a collection of identical agents with a very minimal level of state properties; a power-law signature with associated hub effects seems unthinkable in our context. However, we nonetheless observe that some specific agents do "attract" much more attention than others with indegrees of 15 and above (Fig. 4.8). Finally, it is interesting comparing the structural properties of the SSN based on a topological neighborhood with the ones for a signaling network based on a metric distance, see Sect. 3.2.2. Both interaction distances lead to similar levels of clustering and similar average shortest connecting paths. The central difference between the two groups of SSNs lies with the fact the topological SSN is a directed network, while the metric SSN is undirected. As a direct consequence of that, the outdegree distributions of both types of SSNs are fundamentally different: the outdegree of the topological SSN is constant and equal to k, while the outdegree of the metric SSN is identical to the indegree distribution, which was found to be Poissonian-like.

4.3.6 Resilience of Swarming

The effects of noise on swarming in the physical space has been thoroughly discussed in Chap. 3: with discussions about phase transitions, critical exponents, scaling laws, and collective bifurcations. However, a full account of the effects of noise on the swarm signaling network is probably as insightful as it is in the physical space. To this aim, we consider a swarm of $N = 1{,}000$ agents with different numbers k of topological neighbors. Also, we fix the noise level at $\eta_0 = 0.1$, which falls into the range where $\langle\varphi\rangle$ is significantly influenced by the outdegree k [34]. At the very beginning, prior to any interaction, the SSN is strongly connected for $k = 7$ and $k = 10$ and it forms a single giant strongly connected component (GSCC) as shown in Fig. 4.9 (top row). On the contrary, for $k = 3$ the SSN is composed of 114 strongly connected components (SCCs) of very many different sizes: ranging from 1 agent to 99 agents. Another informative quantity is the average neighborhood radius for the entire swarm—the neighborhood radius is given by the largest distance separating a given agent and its k nearest neighbors. The initial average neighborhood radii are 0.78, 1.22, and 1.49 for k equals to 3, 7, and 10, respectively. We then let this complex system evolve through local interactions of the agents and after a long-enough transient, the collection of agents yields vastly different emergent behaviors in both the physical and network spaces as shown in Fig. 4.3.

For the low outdegree $k = 3$, we observe a large number of clusters of locally aligned agents; no large-scale emergent coherent alignment is achieved. This is clearly noticeable in both the physical and network spaces (Fig. 4.3). The average topological radius fell sharply from 0.78 to 0.21 which is consistent with the physical clustering. Furthermore, the dynamics has amplified the fragmentation of the SSN, which, after the transient, contains 267 SCCs of much smaller sizes: ranging from 1 agent to 22 agents (Fig. 4.9a). Note that the number of SCCs for $k = 3$ tends to reach an asymptotic plateau about the value 250 with very small-amplitude fluctuations after approximately 2,000 nondimensional time units. We qualify this regime

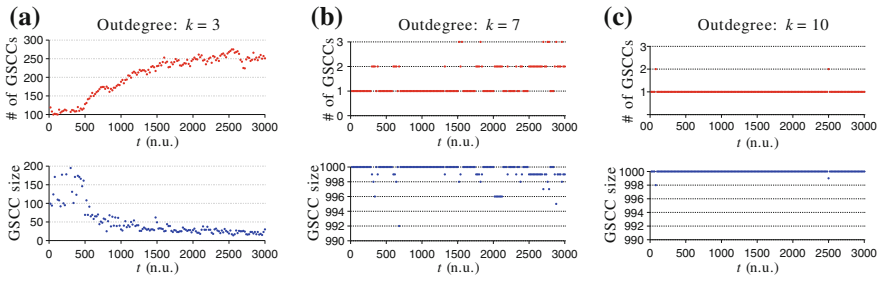

Fig. 4.9 Dynamical properties of the giant strongly connected components (GSCCs) making the SSN. A dynamic range of 3,000 nondimensional time units (n.u.) is considered with $N = 1{,}000$ agents evenly distributed and all initially aligned with the North direction. The noise level is fixed and set to $\eta_0 = 0.1$. *Top row* total number of SCCs. *Bottom row* size of the GSCC found in the SSN. **a** Outdegree $k = 3$; **b** outdegree $k = 7$; **c** outdegree $k = 10$

as quasi-steady state. On the contrary, for both $k = 7$ and $k = 10$, a large-scale coherent alignment is achieved while the distribution of agents is nonuniform but not as physically clustered as in the case $k = 3$. Those observations are corroborated by the fact that the SSN remains as a single giant strongly connected component—apart from very few agents splitting away from the "peloton" (Fig. 4.9b, c)—with almost unchanged average topological radii of 1.16 and 1.44 for $k = 7$ and $k = 10$, respectively. Furthermore, with a much larger value of the outdegree, $k = 40$, the swarm exhibits a higher level of resilience to noise with quite different variations of the alignment with the noise level as compared to other smaller values of k considered.

4.3.7 Controllability of Swarming

If one wishes to control the dynamics of collective behaviors—a goal of tremendous importance for both natural and artificial swarms, we now know that it is necessary identifying the swarm's architecture, in other words the SSN. The importance of controlling or driving a swarm is better understood by revisiting two biological systems such as a flock of birds or a school of fish. For instance, evasive maneuvers triggered by a predator approaching or by collision avoidance, and are collective responses induced by one or a few agents perceiving the threat and responding to it. Those agents are said to be informed since they involuntarily have a privileged access to out-of-the-swarm informational signals. Moreover, these few agents effectively are driver agents: they are able to control the entire swarm by bringing the other agents to swiftly respond to a threat that they are not directly detecting. It is worth adding that those driver agents do not possess any "super" power of any sort but they simply temporarily become informed "leaders" as they happened to have discerned the danger first; any other agent in the swarm could be driving the group as long as it is subjected to specific external cues which are not made available globally to the whole swarm. Therefore, controllability is a vital factor for a swarm to robustly and effectively perform a dynamic collective response benefiting the majority of the group members.

As we will see in Chaps. 5 and 6, from the engineering control viewpoint, a dynamical system is said to be controllable if it can be driven from any initial state to any desired final state in finite time. Recently, the field of complex networks has seen the emergence of new general theories and tools related to the controllability of such networks. The two most prominent controllability tools are: (i) *the structural controllability* framework developed by Liu et al. [35], and (ii) *the exact controllability* framework very recently introduced by Yuan et al. [36]. In applying both the exact and the structural controllability tools, one has access to the details of the extent of the swarm's controllability [37].

First, it is necessary identifying the set of agents that, if driven by different signals, can offer full control over the SSN. To gain full control over a directed network, it is necessary and sufficient to directly control each and every unmatched node—a node is said to be matched if a link in the maximum matching points at it; otherwise it is

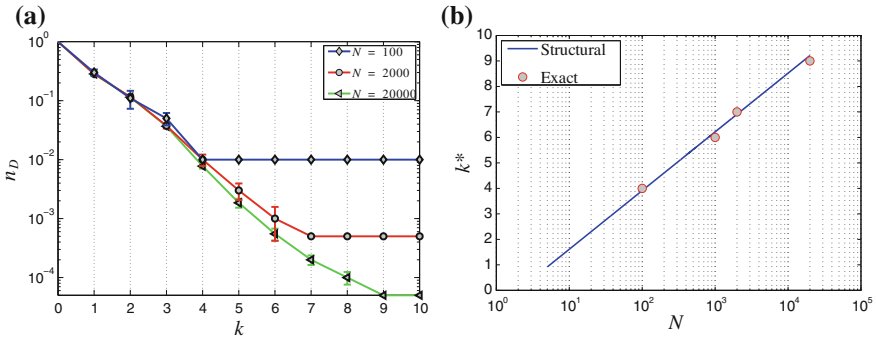

Fig. 4.10 **a** Density of required driver agents for a swarm with topologically interacting members versus the number of neighbors (k) for three different swarm populations (N). Results applying the exact controllability tool were collected for 10 distinct SSNs at each data point. The average density of driver nodes is calculated and the related standard deviations are illustrated by means of *errorbars*. **b** Required number of topological neighbors (k^*) in a swarm to reach full controllability versus swarm size (N). The *blue line* corresponds to the approximate analytical result from the structural controllability analysis. The *red dots* refer to the result obtained with the exact controllability tool

unmatched—and there are directed paths from the input signals to all matched nodes. In our swarming framework (see Sect. 3.2), interactions among agents are either "on" or "off" depending on whether the pair of agents is topologically interacting or not—or equivalently we can say that the weights of the constituent links are binary numbers, 0 or 1. This highlights the fact that link weights are not free-independent parameters. Hence, the exact controllability framework appears suitable [36]. Figure 4.10a shows the results of the exact controllability analysis applied to the dynamical swarm at any given point in time. One can see that the number of driver nodes decrease exponentially as k increases. One can conclude that if the number of nearest neighbors reaches a value around 6 to 8—typical values for the number of topological neighbors observed by Ballerini et al. [16] during field experiments with bird flocks—every agent not only affects and is affected by all other agents within the group, but more importantly, is capable of full control over all other agents, i.e., the swarm.

One concern that should be addressed regarding the above results on the number of driver nodes and the overall controllability of the swarm is associated with the dynamic nature of the SSN. Since the SSN is intrinsically a switching network—at each instant a certain number of links are broken while the exact same number of edges are created due to the motion of the agents in the physical space—one can prove that it is controllable at each instant, assuming of course a high-enough value for k, for example, around 6 to 8. If that is the case, it is known from control theory associated with dynamic hybrid systems that the overall switching dynamical system is controllable [38]. However, if the value of k is not large enough to have a controllable swarm at each instant, then this analysis reveals a lower bound for the control centrality of each single agent, i.e., the ability of a single agent to control the whole swarm [39].

In either natural or artificial swarms, it is more realistic to have nonbinary weights for communication links in order to model the imperfection of the information transfer channel. Thus, it is necessary to consider how the swarm controllability is affected by changing the weights of edges of the SSN. Moreover, such a study would reveal the efficiency of our simple model in analyzing the swarm controllability associated with realistic cases. To that end, a structural controllability analysis of the swarm is performed. A system's structural controllability is to a great extent encoded in the underlying degree distribution, $p(k_{\text{in}}, k_{\text{out}})$. That is, the number of driver agents is determined mainly by the number of incoming and outgoing links each node of the SSN has, and is independent of where those links point at [35]. As mentioned before and by construction, the outdegree distribution of the SSN is a Dirac delta distribution, while its indegree distribution very much resembles the one of a k-nearest random digraph [34], namely a Poisson distribution associated with mean degree k. To allow for an analytical study of the structural controllability of the swarm, we therefore consider the following degree distributions:

$$\begin{aligned} p_{\text{out}}(k_{\text{out}}) &= \delta(k_{\text{out}} - k), \\ p_{\text{in}}(k_{\text{in}}) &= \frac{k^{k_{\text{in}}}}{k_{\text{in}}!} e^{-k}, \end{aligned} \tag{4.3}$$

which lead to the number of driver agents at each time instant being given by $N_{\text{D}} \approx \frac{N}{2} e^{-k}$, in the large k limit [34]. Figure 4.10b shows the required number k^* of topological agents to achieve full controllability of the swarm based on this analytical result. In other words, Fig. 4.10b provides an answer to the following question: for a given swarm population N, what is the number of topological neighbors k^* required to confer to each and every single-agent full controllability "powers" over all other agents. Note that this result based on the structural controllability is in very good agreement with those obtained using the exact controllability framework.

The last question that should be answered regarding the above result on the number of driver nodes and the overall controllability of the SSN lies with the dynamic nature of the SSN. Since the SSN is intrinsically a switching network—at each instant a certain number of links are broken, while the exact same number of edges are created due to the motion of the agents in the physical space—one can prove that it is controllable at each instant, assuming of course a high-enough value of k. If that is the case, it is known from control theory associated with dynamic multiagent systems that the overall switching dynamical system is controllable [38].

4.3.8 Swarm Network Dynamics

A central point to always keep in mind is the fact that the SSN has a dynamics that is evolving hand in hand with the dynamics of the agents themselves. Hence, the connectedness and the structural properties of the SSN are in general not constant. There is a profound connection between, on the one hand, the dynamics of the

collection of agents in the physical space and the structural properties of the SSN as well as its own dynamics, on the other hand. This important note is well reflected in Fig. 4.3, which stresses the parallel between the structure of the swarm in the physical space and the associated SSNs for the three different values of the indegree considered, namely $k = 3, 7$, and 10. The network nodes are exactly located at the agents' physical locations, and the directed links are colored according to the value of the indegree, k_{in}, of the source node from which they are originating. For instance, we are able to visually correlate high values of the indegree k_{in} to small radii of the topological distance.

Another point has to be made about the connection between SSN structure and swarm dynamics in terms of consensus speed. Intuitively, one can easily imagine that a larger number of topological numbers k leads to faster consensus since the connectivity of the network underpinning the dynamics of the interacting swarming agents affects profoundly the consensus capability—in general, higher degree of connectivity yields higher rate of convergence to consensus [40]. This fact has very recently been proved exactly by Shang and Bouffanais [41]. However, it is important to note that adding more edges by increasing the number of topological agents with whom one is interacting is feasible but only up to a certain extent as there is always a cost associated with information exchange and also due to inherent limits in terms of signaling mechanisms, sensory, and cognitive capabilities—for instance, see Ref. [42] for such biological considerations with pigeons. This issue will be revisited in Chap. 5.

4.4 Design of Signaling Network for Artificial Swarming

Social transmission of information is critical to the effectiveness of emergent swarming behaviors as we will see in the subsequent chapter. With the signaling network of interaction accessible, the effectiveness of swarm dynamics can directly be apprehended and improved from the angle of signaling network design. The ultimate goal when designing such swarming systems is to mimic some fundamental principles of collective animal behaviors with the objective to autonomously perform specific tasks. Such artificial swarms readily offer tremendous opportunities since they are freed from a large number of "constraints" inherent to biological systems: e.g., short-range signaling mechanisms, obstructed line of sight, etc. It therefore appears that one of the keys to achieving successful artificial swarm designs lies with an effective design of the SSN. However, in practice this is easier said than done given the lack of sound design principles in the form of relationship between parameters measuring the effectiveness of some specific swarming behaviors and canonical structural properties of networks for the SSN: e.g., clustering coefficient, shortest connecting path, degree distribution, centrality, connectedness, algebraic connectivity, etc. [22].

In this section, we introduce a framework for analyzing such design principles for one of the most prevalent collective decision-making processes consisting in achieving global consensus—consensus means the convergence to a common state

asymptotically or in a finite time among all group members through local interactions. To the best of our knowledge, no clear relationship between the structural properties of the SSN and speed to consensus has ever been presented. Moreover, in their review paper, Arenas et al. [43] mentioned the significant discrepancies in results for different network models when considering the related problem of synchronization in complex networks. According to Arenas and his co-authors, these discrepancies originate from studies where multiple nonindependent parameters characterizing the network were concomitantly changed. This stresses the difficulty in carrying out thorough parametric studies on such networked systems. Indeed, in general, all canonical structural properties of a network bear a certain level of interdependence. Here, the thorough study of distinct network models—two undirected and one directed—allows us to establish some clear relationships between clustering coefficient, shortest connecting path and speed to consensus. All three network models use a unique control parameter, different for each model, enabling us to investigate a large interval of values for the clustering coefficient and shortest connecting path. These models highlight the known interdependency between *CC* and *SP*, but here in a particular framework, since swarm signaling networks are intrinsically constrained to have more or less a fixed average degree.

4.4.1 Models of Signaling Networks

Let us consider three families of complex networks generated through three distinct one-parameter algorithms for which we have an extensive indirect control over the value of the clustering coefficient (*CC*) and shortest connecting path (*SP*). The first two families yield complex undirected networks, while the third one is designed to generate complex directed networks. The first model is the well-known Watts and Strogatz (WS) model where it is possible to manipulate *CC* and *SP* by changing the control parameter p, the probability of rewiring randomly each edge—making the network go from a totally ordered network to a random one [4]. The second model is based on an algorithm introduced by Holmes and Kim (HK) to grow a scale-free network with a tunable clustering coefficient [44]. This HK model is modified to ensure that the ratio between the number of nodes and edges, i.e., the degree, remains constant on average, when graphs with the same number of nodes are generated. This modification of the original HK model is suggested by the properties of the SSN we presented in the previous sections, as well as to avoid pitfalls described by Arenas et al. in their review paper Ref. [43], in relation with parametric studies on complex networks. It is worth adding that the WS model intrinsically has a constant agent (i.e., vertex or node) to edge (i.e., link) ratio—in other words, it has a constant average degree—just like our modified Holme and Kim (MHK) model. Hence, these two undirected one-parameter models can be considered as rewiring algorithms of an original model under constraint—this constraint being the constant average degree. By construction, both the WS and MHK models produce undirected graphs similarly to metric-based SSNs. As already mentioned in Sect. 4.3.1, using

a topological neighborhood of interaction rule such as the k-nearest neighbor rule, however, yields directed networks. Thus, a third one-parameter model is introduced. Based upon the topological neighborhood rule, this third model is designed to fix both the average indegree and the outdegree regardless of the value of the control parameter. Using this so-called modified topological neighborhood of interaction (MTNI) model, it is found possible to establish a reciprocal mapping between the outclustering coefficient and the control parameter.

To analyze the impact of the clustering coefficient on speed to consensus for undirected networks, the MHK model is introduced and compared with the WS model. Both models provide a tunable clustering coefficient by controlling one single parameter at constant average degree. Interestingly, both MHK and WS models yield widely different degree distributions thereby allowing us to further assess the influence of the degree distributions on the consensus reaching process. The MHK model is inspired by an algorithm proposed by Holme and Kim [44] devised to design scale-free networks with a tunable clustering coefficient. It can readily be described by the following simple steps:

1. Create four nodes;
2. Add edges so that each node is connected to exactly two other nodes;
3. Randomly link one of the existing nodes to a newly added one;
4. Given a probability P either: (i) add an edge between the new node and another node in the network that increases the number of triangles (in the clustering coefficient sense), or (ii) add an edge that does not increase the number of triangles;
5. Repeat from step 3 until the desired number of nodes N is attained.

A graphical representation of the above successive steps is shown in Fig. 4.11.

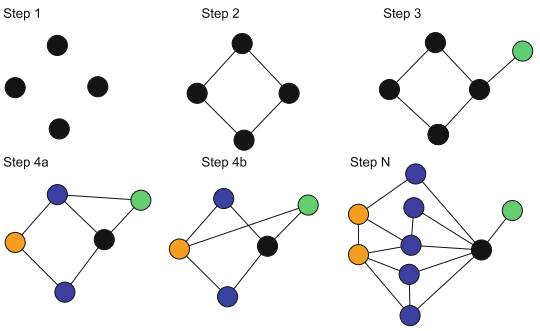

Fig. 4.11 Illustration of the successive steps in the MHK model. A *black node* represents an unchanged node at any given step, the *green node* is the newly added node at step 3. At step 1, four nodes are created. At step 2, the nodes are linked together so all nodes have exactly two neighbors. At step 3, the *green node* is added and attached at random to one of the four *black nodes* in the existing network. At step 4a, an edge is added between a *blue node* and the *green one* to increase the overall clustering coefficient, while at step 4b, an edge is added between an *orange node* and the *green* one thus decreasing the clustering coefficient. Step N shows how the network might look like after a couple of iterations with a new *green node* that has to be attached either to increase or decrease the clustering coefficient

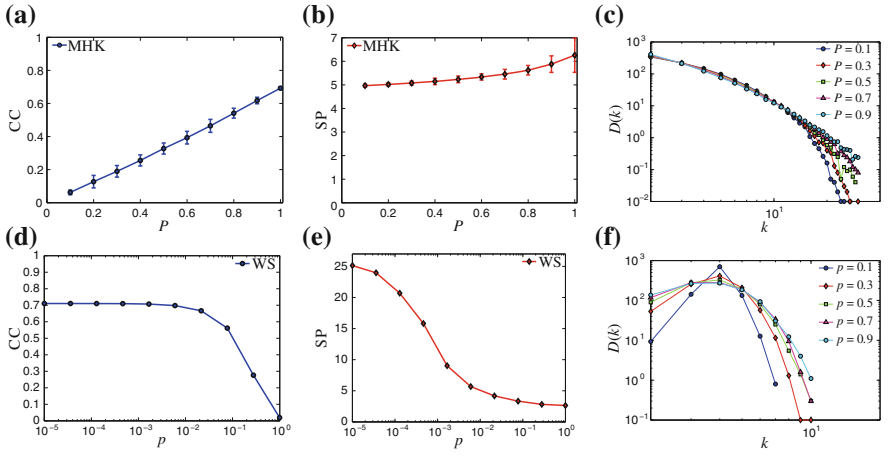

Fig. 4.12 *Left column* Clustering coefficient, *CC*; *Central column* Shortest connecting path, *SP*; *Right column* Degree distribution $D(k)$, for varying values of the probability P (resp. p) in the unit interval. *Top row* for the MHK model, each data point is obtained by averaging over a statistically ample enough sample comprising 100 networks, each having $N = 1{,}000$ nodes and $E = 3{,}992$ edges. The *errorbars* represent the standard deviations to the average values. *Bottom row* for the WS model, each data point is obtained by averaging over 50 networks, each having $N = 1{,}000$ nodes and an average degree $\langle k \rangle = 4$

The one-to-one relationships between control parameters—P for the MHK model and p for the WS model—and clustering coefficient or shortest connecting path are shown in Fig. 4.12. Interestingly, for the MHK model, the variations of the clustering coefficient are found to be practically linear with P, while the shortest connecting path increases extremely moderately in comparison with the same results for the WS model. Specifically, for each value of the probability P, the average clustering coefficient and shortest connecting path and the associated standard deviations are calculated using a statistically ample enough sample of 50 networks generated from the MHK model with $N = 1{,}000$ nodes and $E = 3{,}992$ edges. The constant average degree is thus equal to the fixed degree $\langle k \rangle$. By construction, an increase in the value of the probability P leads to an increase in the clustering coefficient. The results for the WS model are identical to those originally reported by Watts and Strogatz, with here $N = 1{,}000$ and an average degree $\langle k \rangle = 4$.

Using the MHK model provides us with a very good testbed to analyze network properties since it is possible to keep the number of agents and number of edges constant while changing other properties of the network—e.g., clustering coefficient, shortest connecting path, degree distribution, etc. The clustering coefficient can be readily and continuously tuned within the interval $[0, 0.7]$, simply by varying the parameter P in the unit interval. The networks have an average degree $\langle k \rangle$ tending toward 4 as the number of nodes N increases. Indeed, the average degree of a network grown using the MHK model is $\langle k \rangle = (8 + 4(N-4))/N$. It is therefore impossible to control the variations of the average degree as can be done using the WS model. In the particular case such that the number of swarming agents is $N = 1{,}000$, the average degree is $\langle k \rangle = 3.992$. To allow for a quantitative comparison of the MHK and WS

models, all graphs generated using the WS model have therefore been chosen to have an average degree of 4. As can be seen in Fig. 4.12, the MHK model is able to produce networks with fat-tailed degree distributions widely distinct from the homogeneous distribution prototypical of the WS model, while having almost the same average degree. It is often argued that the degree distribution is key to many global outcomes, such as stability [45], consensus reaching [46] and controllability [34, 35, 37, 39]. There is no doubt that the significant difference in degree distribution of the MHK and WS models highlights a fundamental difference in the underlying structure of the respective networks.

The flow of information has been shown to be directed in many kinds of natural and artificial multiagent networked systems: bird flocks, fish schools, and wireless sensory networks to name a few [47]. It is therefore essential to extend our investigation to cases involving directed networks. Before going any further with our analysis of directed SSNs, it is worth stressing some known and yet important differences between directed and undirected networks. When considering undirected graphs, the clustering coefficient is defined using triangles made of undirected edges in the graph [22]. However, in the case of directed graphs, four distinct types of clustering coefficients can be considered depending on how triangles are formed out of directed edges [33]. Following the definitions and terminology of Ref. [33], we consider the clustering coefficient out, CC_{out}, as neighbors of each node in our directed graphs are pointed at with outward edges, thus being compatible with what has already been applied in Refs. [34, 45]. Moreover, the degree of a node also needs to be specified in a different way—for undirected graphs the in- and outdegree are identical which is generally not the case with directed networks. Here, it is only the indegree distribution that is examined since the outdegree is constant and equal for all nodes given that our model is based on the k-nearest neighbor rule to represent the topological distance. With these differences in mind, we propose a modified topological neighborhood of interaction (MTNI) model that is based on a directed signaling network. Similarly to the case based on undirected networks in the framework of the MHK and WS models, we aim at investigating the influence of some adequately controlled network properties on swarm dynamics in terms of consensus reaching.

The MTNI model is a one-parameter stochastic model devised to allow for the tuning of the clustering coefficient (out) by changing the probability $\mathscr{P}$ of choosing a neighbor at random versus a nearest neighbor in the topological sense. Specifically, at the core, the MTNI model is based on the topological neighborhood distance observed in flocks of starlings [16]. The MTNI model can be split up into two parts, the model studied in Sect. 4.3 and a random rewiring. The model works by first randomly distributing nodes on a 2D plane. Each node is then given a fixed number k of neighbors to point at, where each neighbor with probability $1 - \mathscr{P}$ are picked from the set of k-nearest neighbors based on the smallest Euclidean separation distance. In the extreme case where $\mathscr{P}$ is 0, the model generates k-nearest neighbor networks [34]. Conversely, when $\mathscr{P} = 1$, the SSN is a pure random directed regular network with a fixed outdegree k.

Interestingly, a one-to-one relationship between the clustering coefficient and the control parameter $\mathscr{P}$ is also uncovered for the MTNI model, similarly to what was

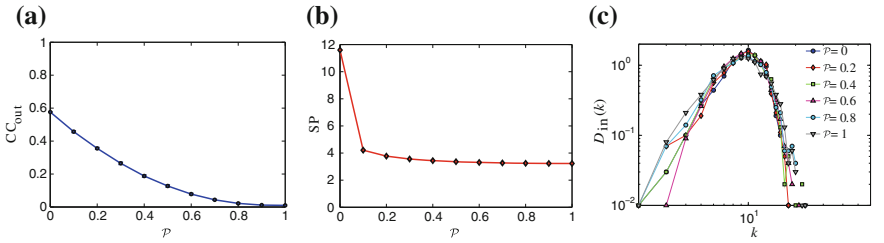

Fig. 4.13 *Left* Out clustering coefficient, CC_{out}; *Center* Shortest connecting path, *SP*; *Right* Indegree distribution, for different values of the probability $\mathscr{P}$ in the unit interval; $\mathscr{P}$ being the control parameter of the MTNI model. Each data point is obtained by averaging over a statistically ample enough sample comprising 50 networks, each having $N = 1{,}000$ nodes and $k = 10$ neighbors, i.e., a fixed outdegree of 10

previously obtained with the WS and MHK models. The shortest connecting path decreases monotonously with $\mathscr{P}$ with extremely moderate variations above $\mathscr{P} = 0.2$. This can easily be seen in Fig. 4.13a, b. This first result is of importance as it proves that we have full control over the clustering coefficient through the tuning of $\mathscr{P}$. However, the upper limit on CC_{out} is approximately 0.6 and corresponds to $\mathscr{P} = 0$, i.e., for the pure topological model. This result is in total agreement with those shown in Sect. 4.3.4. The MTNI model therefore has the advantage of producing networks where the agents have a constant outdegree and constant average indegree, while CC_{out} can continuously be changed within the interval [0, 0.6]. The impossibility to generate CC_{out} beyond the upper limit of 0.6 does not prove to be an issue since networks possessing high-clustering coefficients tend to be quite ineffective at global consensus reaching. On the contrary, small values of CC_{out} are sought when the emphasis is put on the effectiveness of achieving global consensus.

Let us now turn to the indegree distributions associated with the MTNI model with varying control parameter $\mathscr{P}$. The case $\mathscr{P} = 1$ is straightforward as it is a purely random network, we expect a normal degree distribution centered about the average degree, which is equal to the constant outdegree. At the other end of the unit interval, the case $\mathscr{P} = 0$ corresponds to the pure topological neighborhood model, which was shown to lead to a Poissonian-like distribution, see Sect. 4.3.5. In the present case, we consider the particular case $k = 10$ and with such a relatively large outdegree, the Poissonian-like distribution reduces to an almost Gaussian one. Four other values of $\mathscr{P}$ have been considered and the associated indegree distributions are shown in Fig. 4.13c. One observes that with the outdegree $k = 10$, the MTNI model produces networks with practically Gaussian distribution for all values of the control parameter $\mathscr{P}$.

4.4.2 Enhanced Swarming Behaviors

The speed of convergence to consensus is classically assessed by means of the spectral properties of graph Laplacians [22]. Specifically, the second smallest eigenvalue of the graph Laplacians λ_2—a.k.a. algebraic connectivity—quantifies that speed of

convergence in the presence of a static graph. With the MHK, WS, and MTNI models introduced, we are now able to directly measure and quantify the speed to consensus for families of SSNs corresponding to both undirected and directed information exchanges with variable clustering coefficient, shortest connecting path, as well as other network properties.

For the spectral analysis of undirected graphs the normalized graph Laplacian [46], defined as

$$\tilde{\mathbf{L}} = \mathbf{D}^{-\frac{1}{2}}(\mathbf{D} - \mathbf{A})\mathbf{D}^{-\frac{1}{2}}, \tag{4.4}$$

is used, with $\mathbf{A}$ and $\mathbf{D}$ being the adjacency matrix and the degree matrix, respectively. Using $\tilde{\mathbf{L}}$ has the advantage of being similar to using the dynamical matrix describing a swarm whose agents have metric local interactions. This is aligned with our goal of designing SSNs in order to generated effective swarming behaviors.

In the previous section, we saw how the WS and MHK models allow us to generate networks with varying *CC* and *SP* through the tuning of control parameters. The procedure is here applied again but now to obtain the second smallest eigenvalue λ_2 of $\tilde{\mathbf{L}}$. By doing so, we are able to plot the variations of λ_2 as a function of *CC* and *SP*. Note that for all three models, the clustering coefficient is bounded within the intervals found earlier. The variations of λ_2 in terms of *CC* and *SP* are obviously represented by a line of points corresponding to given values of the control parameters p and P. These variations are shown in Fig. 4.14a–d for both undirected models.

First, we note that when varying the control parameters, the changes in clustering coefficient and shortest connecting path are quite different for the WS and MHK models. Very little variation of the *SP* is observed with the MHK model for *CC* varying between 0 and approximately 0.7. As is well known, the WS model yields significant variations of the *SP* with the *CC* [4]. Furthermore, it is observed in Fig. 4.14a–d that λ_2 tends toward the same value for networks designed with the two different models when the respective clustering coefficient and shortest connecting path converge toward the same values, even though we have found the degree distributions to be widely different for both the WS and MHK models. This result is noteworthy as it contradicts the frequently encountered statement that the degree distribution is key to many global outcomes of dynamic networked systems. It is worth recalling here that the values found for λ_2 depend on the number of agents, i.e., the number of nodes N of the SSNs.

Using the WS model, it can be shown that λ_2 does not necessarily increase when the shortest connecting path is decreased as can be seen in Fig. 4.14e, f. However, there seems to be a clear relationship between the *SP* and λ_2 when *CC* is kept constant. That observation suggests that λ_2 can be predicted if *CC*, *SP*, and number of agents are known. Reducing *CC* while keeping *SP* constant or nonincreasing is thus found to increase the speed to consensus.

Before testing if a similar relationship with directed networks exists, we need to define the concept of Laplacian matrix for directed graphs. The Laplacian matrix has to be redefined because the in- and outdegree of a node need not be the same in the case of a directed graph, thus preventing us from using the previously introduced

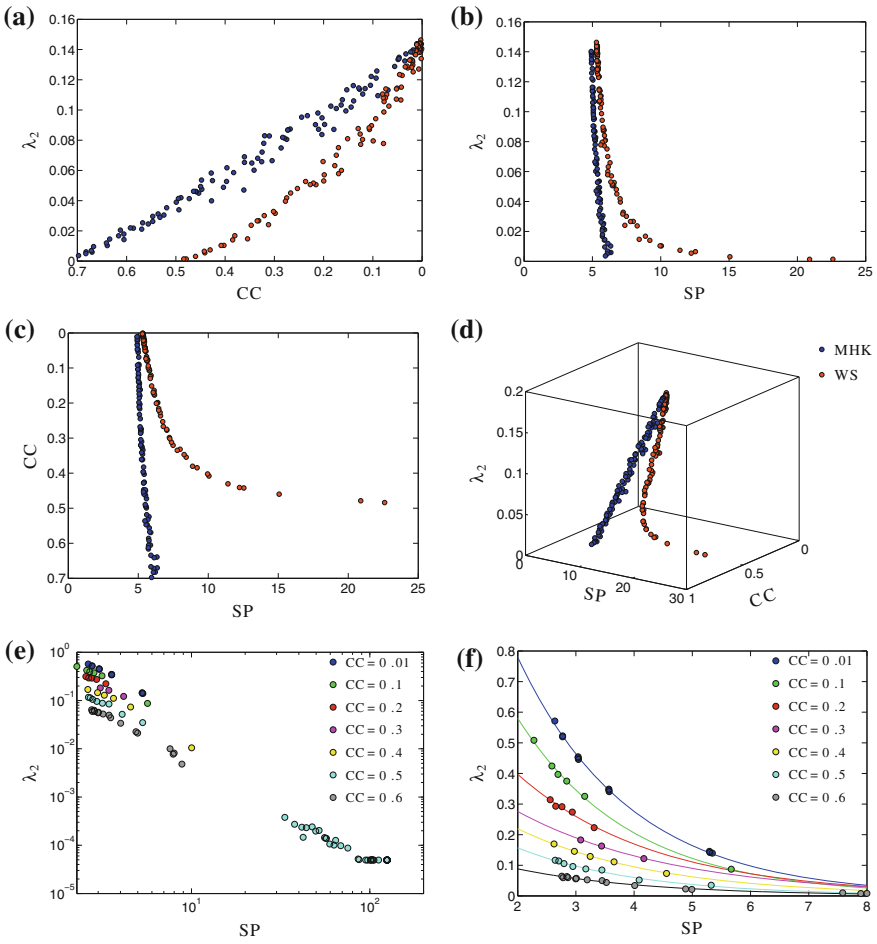

Fig. 4.14 **a–d** Speed of reaching consensus measured using the most significant pole (λ_2). Each data point represents a network designed either using the WS model (*red*) or the MHK model (*blue*). All networks considered possess $N = 1{,}000$ nodes and an average degree of 4 (resp. 3.992) for the WS (resp. MHK) model. **e–f** Variations of the algebraic connectivity with the shortest connecting path for different values of the clustering coefficient for small-world networks generated from the WS model. The average degree varies between 4 and 36 leading to a nonconstant average degree. **e** log–log scale; **f** linear scale with the *colored lines* being exponential fits corresponding to a specific *CC* with an error margin of $\pm\, 2\,\%$. **g–j** Speed of reaching consensus measured using $\mathrm{Re}(\lambda_2)$. Each data point represents a network designed using the MTNI model, with $\mathscr{P}$ varying between 0 and 1. All networks considered possess $N = 1{,}000$ nodes and a fixed outdegree of 10

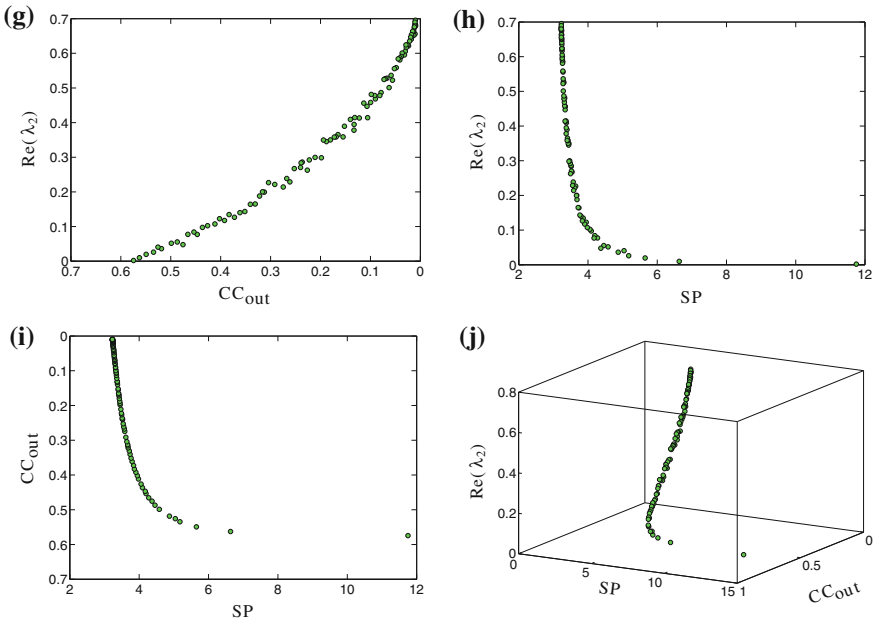

Fig. 4.14 (continued)

normalized Laplacian graph. Thus, we use the following definition for the Laplacian matrix:

$$\mathbf{L} = \mathbf{D}_{\text{out}}^{-1}(\mathbf{D}_{\text{out}} - \mathbf{A}_{\text{out}}), \tag{4.5}$$

where $\mathbf{A}_{\text{out}}$ and $\mathbf{D}_{\text{out}}$ are the out-adjacency matrix and the outdegree matrix, respectively. This is indeed an appropriate choice as neighbors are pointed at with outward edges in the directed SSN. Following the method used for undirected networks, a plot of the variations of the real part of λ_2, *SP* and CC_{out} is shown in Fig. 4.14g–j. An outdegree of 10 is chosen so that the generated networks are strongly connected with $N = 1{,}000$ nodes [34]. We recover a trend similar to the one observed previously with undirected networks; namely the speed to consensus as measured by $\text{Re}(\lambda_2)$ monotonously increases with a decreasing CC_{out} for nonincreasing *SP*. This trend emphasizes the following very important design principle that it is not only important to reduce the *SP* of a network, but the clustering coefficient also needs to be reduced, in order to speed up the global consensus reaching process [48]. Note that this statement is consistent with the result by Xu and Liu [6] showing a clear relationship between spread of information in social networks and the clustering coefficient.

4.4.3 Some Words of Caution

As we have seen throughout this chapter, networks are an extremely powerful tool having a profound impact not only on our understanding of collective behaviors,

but also on our ability to engineer and effectively use complex networks to design artificial swarming systems.

There is no doubt that this relatively new line of reasoning for swarms provides many useful insights. However, networks by themselves are not the panacea to break new ground in understanding and designing swarming systems. At this stage, I would like to offer some words of caution regarding the following items:

- *Swarms are dynamic and need information.* To achieve specific collective behaviors, sufficient and reliable information flow needs to take place throughout the entire swarm, and that in the presence of dynamically changing communication topologies. Hence, the network approach introduced in this chapter needs to be supplemented with an analysis grounded in information theory. The next chapter is concerned with such interplay between network and information in the particular case of collective behaviors.
- *Swarms are dynamic and need computation.* The signaling network influences the dynamics, but this in turn reshapes the network, in a feedback loop. Therefore, a pure network-theoretic approach is going to be insufficient to design robust and effective swarming systems. Several promising ways of coping with the problem of coupling network topology and dynamics are offered by the emerging field of networked control systems.

After all, the swarm signaling network is an adaptive network and its study is known to be very interdisciplinary in nature.

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Chapter 5
An Information-Theoretic Approach to Collective Behaviors

Swarming agents are interconnected organisms or agents. As is well known in our information age, a key benefit of being connected is access to information. We have discovered and observed this interconnectivity from the biological standpoint in Chap. 2, then analyzed it from the physical viewpoint in Chap. 3, and thoroughly studied its network structure and dynamics in Chap. 4. Such dynamic interconnectivity serves the purpose of channeling information exchanges, which are critical to the effectiveness in swarming. Indeed, it is well known that collective animal behavior is dependent on the existence of communication channels enabling information exchange between individuals [1]. For instance, the collective surveillance against oncoming threats of a flock of birds provides a higher level of vigilance only if the information obtained by each pair of eyes is shared among the flock. However, up to now our discussion about information was limited to generalities and we have not detailed the following: (i) the role of information in collective behaviors, (ii) how information is communicated throughout a swarm, and (iii) how information is processed or computed in a decentralized way by the swarm. This chapter is concerned with (i) and (ii), while the next one will deal with distributed information processing from the perspective of decision-making or collective computation.

5.1 Social Information Transmission

Swarm dynamics is governed by interagent interactions and also interactions between individual agents and their environment. As was discussed in Sect. 2.4.1, through all these interactions a social transmission of information occurs, eventually leading to a wide range of self-organizing natural behaviors. Let us come back to the example of the evasive maneuver performed by a flock of birds confronting a threat. The predator is external to the flock and can be said to be part of the environment. The detection of its presence by birds at a close range amounts to an external signal or

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stimuli—a.k.a. sensory information. The detecting agents swerve in order to move away from this stimuli, thereby creating a signal internal to the flock—behavioral information. This behavioral signal contains information and propagates through the flock at high speed and with little distortion [2]. This latter information transmission process can be said to be responsible for the overall effectiveness of the flock in dodging the predator's attack. Furthermore, a fundamental property of informational exchanges is that they can be processed as we will see in Chap. 6. This stands in stark contrast with collective phenomena purely driven by physical interactions that cannot be ignored, as is the case with most examples reported in Sect. 3.1.3. For instance, birds detecting an environmental threat give utmost priority to sensory information, thereby discarding behavioral signals from neighboring conspecifics. In short, information is a crucial currency for animals from a behavioral standpoint. This is certainly also true for artificial swarms as we will see in what follows.

5.2 Role of Information in Collective Behaviors

One of the most fundamental problems natural swarms face is to maintain conditions suitable to their continued operation, whatever the function of the collective behavior is: surveillance, predator avoidance, foraging, etc. Anticipating our discussion in the subsequent chapter, we know from the field of engineering control that one effective way of maintaining operations comes in the form of feedback loop. In many cases, feedback information is sensory in nature, informing the swarm about its external environment and possible changes to it. Information exchanges are then critical to the execution and effectiveness of a host of collective behaviors such as those introduced in Chap. 2. However, animals are generally faced with uncertain environments and fast-changing circumstances, and, often, their survival critically depends on their ability to swiftly manage and respond to such unpredictable changes in their surroundings.

The benefits associated with collective and cooperative grouping appear to be directly linked to the enhanced ability it gives to swarms in terms of dynamically responding to such uncertain and rapidly changing natural environments [1, 3]. In recent years, there has been mounting recognition that distributed transfer of behavioral information is key to the highly responsive nature of swarms [4–7]. It is becoming apparent that collective intelligence in the form of adaptive behavioral responses relies upon having both accurate and sufficient social information exchanges occurring among interacting units. However, an important question related to that and which is still largely unanswered, is whether animals can individually perceive an improved benefit from the effort expended to create such feedback channels. Unsurprisingly, underlying all information transfer is some form of positive feedback, which we saw in Sect. 2.4.2 was central to the emergence of dynamic patterns of collective behavior.

Such positive feedback mechanism, here spurred by sensory information, induces an amplification of the original signal. This effect is certainly desirable since it is well known that information transmission from agent to agent is in general prone to heavy distortion and rapid damping. Recent results by Cavagna and his team based on flocks of starlings performing collective turns, reveal that information about direction changes propagates across the flock with negligible damping and with a linear dispersion law [2]. These very important results cannot be explained by traditional models of diffusive transport of information. However, a full understanding of the discrete process of information transfer remains elusive. Cavagna and his coauthors use a continuous physical model involving the concept of behavioral inertia that somehow can be related to a delay in reaction time at the agents level [2].

The development of these positive feedback channels has undeniable benefits for the long-range transmission of accurate information throughout the swarm. However, if not properly controlled, the amplified sensory signal could result in a centralized control of the dynamics, thereby depriving the swarm from its ability to self-organize and swiftly respond to other signals associated with changes in its surroundings. Stabilizing negative feedback mechanisms are therefore required to counterbalance the amplification of the external sensory signal. We therefore argue here that effective processes of information transmission in swarms rest upon a dynamic balancing between positive amplification of the external signal and negative feedback of behavioral signals internal to the system. Nonetheless, positive feedback is a powerful mechanism to create structure and induce a dynamic response in swarms.

5.3 Information Flow in Swarms

5.3.1 Quantifying Information

A quantitative and universal definition of information is still not agreed upon. The very existence and relevance of such a definition remains in itself an open question. The best formed theory to date is centered on the maximum amount of information that can be transmitted between a source and receiver over a given noisy transmission channel. This is the theory of communication due to Shannon [8], which holds even if transmission errors caused by noise on the channel occur. Note that Shannon's theory says nothing about the actual information content: i.e., the meaning of the informational exchanges are completely ignored. Without delving into the details of Shannon's information theory, it is worth stressing that Shannon's definition of information was inspired by Boltzmann's definition of entropy. In statistical physics, Boltzmann's entropy is a measure of the lack of order in many-particle systems, and it is therefore very interesting to find a connection between the statistical physics viewpoint described in Chap. 3 and the information-theoretic approach adopted in the present chapter. After all, swarms are characterized by self-organizing behaviors in which order emerges out of disorder.

5.3.2 Dynamics of Information Transfer

Very recently, a growing body of work turned to the study of this problem of *information transfer* within a dynamic collective [2, 4, 5, 9, 10]. Tracing the flow of information and quantifying informational exchanges are key to gaining insight into the functioning of swarms.

Empirical observations of collective turn in flocks of starlings have been reported by Attanasi et al. [2]. The variations of the turning order, or rank, of each individual bird as a function of the turning delay is shown in Fig. 5.1a. Attanasi et al. further showed that the first 5 birds to initiate the turn are closed to one another at the beginning of the event, thus highlighting the spatially localized origin of the triggering behavioral signal. For that specific event involving $N = 176$ starlings, it took approximately less than 0.6 s for the information to flow through the whole flock,

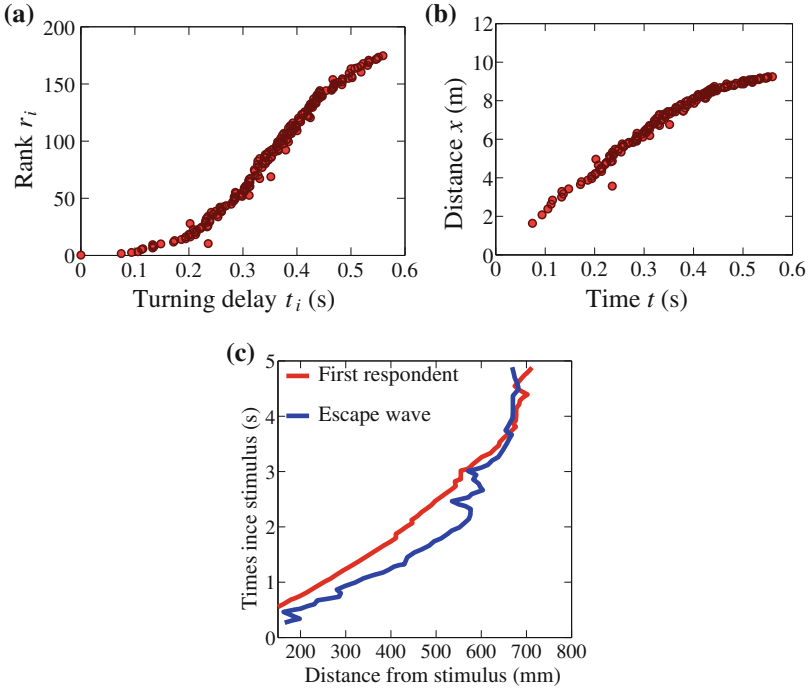

Fig. 5.1 **a** Rank r_i of each bird in a given collective turn event ($N = 176$ birds for event 20110208_ACQ3), that is, its order in the turning sequence versus its absolute turning delay t_i with respect to the first bird to turn. Data and results from Ref. [2]. **b** Distance x traveled by the information as a function of time t ($N = 176$ birds for event 20110208_ACQ3). Data and results from Ref. [2]. **c** Dynamic of information transfer in a school of fish ($N = 51$ fish) startled with a simulated attack. Time since stimulus versus distance from stimulus, for the average position of the first responding individual (*red*) and for the average position of the escape wave (*blue*). Data and results from Ref. [10]

with an information transfer clearly modulated. From the results shown in Fig. 5.1a, Attanasi et al. [2] were able to compute how much distance x the information travels in a time t. The dispersion law they have obtained is presented in Fig. 5.1b and reveals a clear linear regime $x \propto t$—incompatible with a diffusive process of information transfer—during most of the collective turn, with an amazingly fast speed of information transfer of the order of 20–40 ms^{-1} (3 times faster than the flock itself).

Another interesting set of empirical observations has been reported for startled schools of fish by Herbert-Read et al. [10], showing the existence of what Radakov—a pioneer in this scientific quest—called an "escape wave" [11]. Specifically, the authors of this recent study triggered a simulated predatory attack aimed at startling schools of fish placed in an experimental annulus arena. Herbert-Read et al. demonstrate that changes in the direction and speed by a small fraction of spatially localized informed individuals is at the origin of escape waves. In that respect, this fact is in excellent agreement with the results by Attanasi et al. [2] for flocks of birds. They further observed that in the majority of cases, the escape wave passes through the entire school with the dynamics of information transfer being as is shown in Fig. 5.1c. These results are significant because they demonstrate that the escape wave is almost as fast as the first individual fish directly detecting the threat. Both of these results, with schools of fish and flocks of birds, reveal fascinating details about information flow in swarms. Specifically, information is flowing excessively fast and in a nontrivial way throughout swarms.

5.3.3 Transmission Channels

Tracing the flow of information and quantifying informational exchanges are key to gaining insight into the functioning of swarms. It also plays a central role when designing swarming systems. From the engineering standpoint, such effective information transfers highlight the existence of an underlying communication channel that takes the form of the swarm signaling network (SSN) as was seen in Sect. 4.2. Animal collectives use this SSN to effectively respond to changes in the surroundings: e.g., coordinated evasive maneuvers upon detection of a predator or collision avoidance. As emphasized in Chap. 4, the structure and dynamics of swarm signaling networks are quite unique, being temporal and adaptive networks [12] with a dynamics deeply interwoven with the agents' motion dynamics embedded in the physical space. For instance, in the particular case of bird flocks governed by a topological interaction [13], the SSN has been found to be a small-world, homogeneous clustered network whose connectedness is key to yielding resilient swarming behaviors [14]. The knowledge of and access to the structural properties of the SSN revealed the high dynamic controllability of swarms [14, 15]—where few agents are capable of driving the dynamics of the swarm as a whole—as well as very effective consensus reaching processes [16].

Any information exchange—whether through a digital wireless network as in the case of mobile sensory networks, or through a fluid as in the case of flocking birds

and schooling fish—occurs over communication channels that are imperfect due to intrinsic limitations, primarily in terms of channel capacity and topology. For specific classes of networked control systems, necessary and/or sufficient conditions on the smallest data rate [17, 18] and on the communication topology [19, 20] for their stability or stabilizability have been established.

Despite strong similarities between networked dynamical systems and natural swarms, there exist numerous differences in the nature and properties of their respective communication channels. Significant attention has been devoted to understanding these differences in terms of topology and structure. In particular, the emphasis has been put on the impact of dynamic and switching topologies on the collective dynamics of locally interacting agents [16, 19, 21, 22]. This effort is justified by our incomplete understanding of natural swarming behaviors, and also by the ongoing development of new biologically inspired designs of artificially swarming systems. Comparatively, the specificities of the transmission channels of naturally swarming systems in terms of capacity has been relatively overlooked. However, the problem of reduced information flow due to limited data rates in social transmission of information is as critical in the natural realm as it is in the engineering one. For instance, it has been found that the collective synchronization of neurons in dorsal root ganglions is thwarted by a chemically induced reduction of the firing frequency [23], which effectively corresponds to a reduction in collective information flow associated with smaller data rates.

We now turn to the use of the self-propelled particle (SPP) model introduced in Sect. 3.2 to gain insight into exactly how information flows through a swarm. In this class of models, like in real swarms, collective decisions globally emerge from local information exchanges associated with actual interagent interactions. At this stage, it is important highlighting a notable difference between communication in networked dynamical systems and in natural swarms. In the former, communication implies a deliberate transmission, whereas in the latter they are often not associated with deliberate exchanges of information. A study of swarm dynamics benefits from a description and representation of the true communication that follows those information transfers [24]. As already highlighted, any real communication channel, irrespective of its topology and the nature of the signal, has a finite informational capacity owing to its noisiness and limited bandwidth. For natural swarms, channels may consist of chemicals (e.g., chemotactic aggregation of amoebae, colony of ants) or of various forms of energy such as electromagnetic waves and light, sound vibrations, pressure, or temperature. In practice, more than one channel may be operating simultaneously as is the case with fish while schooling.

The network-theoretic approach discussed at length in Chap. 4 opens new avenues for the study of the unifying concept of swarm information flow representing the propagation of behavioral changes. However, such a high-level structural representation should not hide the complexity of a central part of the real informational channel associated with the agents' sensory cascade [24]—detection, processing followed by response—taking place when information hits a node and is routed through the signaling network. This crucial factor can be better fathomed when considering the

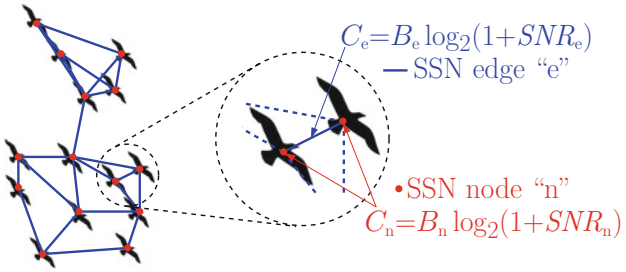

Fig. 5.2 Schematic of a networked flock of birds with the associated transmission channel in the form of the swarm signaling network (SSN). *Edges* represent an interaction between two agents. Nodes are the agents themselves, which act as routers for the behavioral information

prototypical swarming behaviors of predator avoidance and collective turn described in Sect. 5.3.2. These localized behavioral responses initiated by the informed agents constitute a signal transmitted through the medium (edges of the SSN) which, in turn, is detected by the agents (nodes of the SSN) linked to the informed nodes (see Fig. 5.2) if the SSN has the required connectedness. This latter property of the SSN has been shown to primarily depend on the interaction distance—metric, topological, or hybrid [25]—and the density of swarming agents, and to profoundly influence the consensus reaching dynamics [16]. For swarms of topologically interacting agents, we have established in Chap. 4 that the SSN is a homogeneous, small-world, and moderately clustered network. Moreover, the SSN is a temporal adaptive network, whose dynamics is tightly coupled to that of the agents in the physical space. It is therefore necessary to account for some of the functional details of each agent, and in particular, its sensory cascade [24], which can conceptually be modeled using the control-theoretic concept of multi-input and multi-output plant.

5.3.4 Capacity of the Transmission Channel

A full description of the information flow through a dynamic swarm would not be complete without another conceptual layer borrowing elements from information theory. This integrated view of information and control theories has been successful in advancing our understanding of the interplay between control and communication in network dynamical systems [26]. If the total power in a channel is distributed between the signal S and random (Gaussian) noise N, the Shannon–Hartley theorem provides the maximum rate of information transmission, or capacity C of the channel

$$C = B \log_2(1 + SNR), \tag{5.1}$$

where B represents the channel's bandwidth and $SNR = S/N$ is the signal-to-noise ratio [27]. Bandwidth B is technically defined as the range of frequencies

that can be transmitted, but in natural systems the lowest frequency is always zero and it is therefore identified with the highest frequency at which the channel can be varied. In most real channels, the value of the bandwidth cannot always be easily determined. In the particular case of human information exchange through speech, it has been shown that a bandwidth of 7 kHz is required to distinctively understand individual syllables [28]. In the case of swarms, one has to consider C_e associated with informational signaling through the medium—the network edges "e": e.g., the ambient fluid for flocking birds, schooling fish, and synapses for neurons—as well as C_n for the sensory cascade internal to each agent—the network nodes "n", which serve as routers for the behavioral information (see Fig. 5.2).

5.3.5 *Informational Bottlenecks in Collective Behaviors*

The study of the effects of reduced accuracy—owing to the ubiquitous presence of environmental noise combined to limited sensory capabilities—in social transmission of information has received significant attention at both the experimental and modeling levels and was thoroughly discussed in Chap. 3. Comparatively, the consequences of limited social information flow within natural swarms have been relatively overlooked despite the realization of its significance in networked control system theory over a decade ago [17, 26]. Information is often defined as being the capacity to organize a system. This definition resonates very well with our search for informational bottlenecks hindering the self-organization of swarms.

We therefore return to the problem of global information flow through the SSN with the aim to identify the possible limitations associated with this process. The max-flow min-cut theorem [29] informs us about informational bottlenecks, whereby the maximum information flow rate is given by the minimum capacity of the network edges or nodes. In other words, the spread of information through the swarm is either limited by the signaling through the medium (extrinsic limit) or by the agent's sensory cascade (intrinsic limit). At this point, it is worth stressing yet another specificity of natural swarming individuals pertaining to this intrinsic limit to deal with information flow at the agent's level regardless of the nature of the signal and how it propagates information through the surrounding medium. Indeed, for most animals (including humans), there is an enormous gap between the information capacities of sensory organs and the capacity of the central nervous system to analyze and retain information [24]. Consequently, without loss of generality we can associate C_n with the information processing capacity of the agent.

As can be seen from Eq. (5.1), the informational capacity is either bandwidth- or SNR-limited, therefore leading to only four possible distinct informational bottlenecks:

(i) low signal-to-noise ratio, SNR_e, in the medium, which often occurs because of high levels of ambient noise;
(ii) low signal-to-noise ratio, SNR_n, within each agent;
(iii) low bandwidth or data rate, B_e, of the medium;
(iv) low bandwidth or data rate, B_n, in processing information at the agent's central nervous system level.

The influence of low SNR_e in the medium, option (i), can be used to explain empirical evidences of some specific swarming breakdowns: e.g., schools of fish disperse at dusk [30–32]. Essentially, the phase transitions presented in Chap. 3 (see Sect. 3.3) and uncovered using Vicsek's model and its variations [33], from a collectively ordered phase to a disordered one following an increase in ambient noise level, can be traced back to information flow breakdowns through noisy channels: either with low SNR_e [34] as in option (i), or with low SNR_n [35] as in option (ii).

Note that option (iii) is physically irrelevant for natural swarms since in most media, B_e is typically very high [24], unless it is artificially constrained as will likely be the case with man-made swarms. Other empirical evidences are stressing the importance of a sufficiently high bandwidth[1] in complex adaptive systems: e.g., information transmission through signaling relay during the collective migration of social amoebae [36], or the induction of differential anesthesia when chemically reducing the firing frequency of neurons in dorsal root ganglions [23]. These empirical evidences are additional motivations for the investigation of the overlooked option (iv) and the associated study of the effects of information flow breakdowns in swarms stemming from either the finiteness of the agents' bandwidth B_n or an artificially induced reduction in B_n or B_e.

All the above discussion on bandwidths and signal-to-noise ratios might appear quite abstract and remote from the central topic of this book, namely swarms. However, the following facts about ants and ant colonies, I hope, should be convincing of its relevance and significance. To fix ideas, it is interesting knowing that in the particular case of fire ants orientating, a rough estimate of C_n is between 0.01 and 2 bits/s [37]. Moreover, ants are capable of measuring frequencies and this capability is actually used to achieve an effective division of labor within the colony. The size of each group of workers is regulated according to colony needs through some flow of information among ants [38].

5.3.6 *Conditions for the Emergence of Collective Behavior Under Data Rate Limitations*

In the sequel, we focus on the overlooked options (iii) and (iv), and study the effects of information flow breakdowns in swarms stemming from either the finiteness of the

[1] Equivalent to information update since the Nyquist rate $2B = f$ relates bandwidth and frequency of update.

agents' bandwidth B_n or an artificially induced reduction in B_n. Given the Nyquist theorem, to consider the effects of a reduction in B_n is equivalent to considering an increase in the unit interval

$$T_\mathrm{n} = \frac{1}{2B_\mathrm{n}}, \tag{5.2}$$

which is the minimum time interval between condition changes of data transmission signal, a.k.a. the symbol duration time [27]. Note that our analysis and the associated results would still hold if we were to artificially reduce the bandwidth of the medium, $B_\mathrm{e} = 1/(2T_\mathrm{e})$ as is later suggested.

In our minimalistic model, the N topologically interacting SPPs perform a collective behavior of the consensus type for their direction of travel θ_i. At each instant, the collective state of the swarm is characterized by the following global vector of state variables

$$\boldsymbol{\Theta}(t) = [\theta_1(t),\ \theta_2(t), \ldots,\ \theta_N(t)]^T, \tag{5.3}$$

which is updated according to the time update rule

$$\theta_i(t + T_\mathrm{n}) = \theta_i(t) + \frac{T_\mathrm{n}}{k_i} \sum_{j \sim i} \left\{\theta_j(t) - \theta_i(t)\right\} + \eta_\mathrm{n}\xi_i(t), \tag{5.4}$$

where $k_i = k$ is the fixed number of individuals in the topological neighborhood $j \sim i$ of i and $\eta_\mathrm{n}\xi_i(t)$ is a Gaussian white noise ($\xi_i(t) \in [-\pi, \pi]$). Equation (5.4) is identical to Eq. (3.7), albeit with notations adapted to the current framework. In addition, Eq. (5.4) is a discrete-time version of the minimal model consistent with experimental correlations in natural flocks of birds, while also predicting the propagation of order throughout entire flocks [39].

To allow for an analytical study of this system, we first neglect the effects of noise, and given any initial state $\boldsymbol{\Theta}_0$ at time $t = 0$, at any point in time the swarm's state is

$$\boldsymbol{\Theta}(t + mT_\mathrm{n}) = \left[\Pi_{j=1,\ldots,m} \mathbf{P}_\mathrm{n}((j-1)T_\mathrm{n})\right] \boldsymbol{\Theta}_0, \tag{5.5}$$

given the dynamical swarm update

$$\boldsymbol{\Theta}(t + T_\mathrm{n}) = \mathbf{P}_\mathrm{n}(t)\boldsymbol{\Theta}(t) + \eta_\mathrm{n}\boldsymbol{\Xi}(t), \tag{5.6}$$

with

$$(\boldsymbol{\Theta}(t), \boldsymbol{\Xi}(t)) = \left(\{\theta_i(t)\}^\mathrm{T}_{1 \le i \le N}, \{\xi_i(t)\}^\mathrm{T}_{1 \le i \le N}\right), \tag{5.7}$$

and

$$\mathbf{P}_{\mathrm{n}}(t) = (\mathbf{I} - T_{\mathrm{n}}\tilde{\mathbf{L}}(t)), \tag{5.8}$$

are Perron matrices dependent on the unit interval T_{n} [19], with $\tilde{\mathbf{L}}(t) = L(t)/k$, $L(t)$ being the outdegree graph Laplacian for the SSN characterizing the instantaneous communication topology between individuals. It is worth adding that these Perron matrices fully embody the instantaneous relationship between information flow and communication structure at the core of our problem.

In the presence of a static communication topology, the stability of the dynamical system would be governed and controlled by the spectral properties of the constant Perron matrix [19]. In the present case, however, the constantly reconfigurable and switching network requires a generalization of such stability analysis accounting for possibly varying symbol duration times T_{n}. In the absence of noise, the time-dependent networked sampled-data system is simply governed by

$$\boldsymbol{\Theta}(t + T_{\mathrm{n}}) = \mathbf{P}_{\mathrm{n}}(t)\boldsymbol{\Theta}(t) = (\mathbf{I} - T_{\mathrm{n}}\tilde{\mathbf{L}}(t))\boldsymbol{\Theta}(t), \tag{5.9}$$

We have proved in Ref. [40] that a necessary and sufficient condition for this system to be stable, and asymptotically stable, is that it is stable at every point in time $t_j = jT_{\mathrm{n}}$. This key result was obtained by studying the convergence of infinite products of matrices $\mathbf{P}_{\mathrm{n}}(t_j)$ by means of the joint spectral radius. More importantly, it leads to the following sufficient condition, which has very important practical implications, as it guarantees the system's stability, namely the consensus reaching of the swarm under certain conditions on the symbol duration T_{n}. Specifically, a sufficient condition for the stability of the networked sampled-data system (5.9) is given by the following upper bound on the symbol duration time, which has to be verified at every single point in time $t_j = jT_{\mathrm{n}}$:

$$T_{\mathrm{n}} < \frac{2}{\max_{1\le i\le N} |\lambda_i(\tilde{\mathbf{L}}(t))|} \qquad \text{for all } t, \tag{5.10}$$

where $\lambda_i(\tilde{\mathbf{L}}(t))$ are the eigenvalues of $\tilde{\mathbf{L}}(t) = L(t)/k$. Based on condition (5.10) and the relation $T_{\mathrm{n}} = 1/(2B_{\mathrm{n}})$ between unit interval and bandwidth, we find that if the bandwidth satisfies the sufficient condition

$$B_{\mathrm{n}} > B_{\mathrm{n}}^0 = \frac{1}{4} \max_{1\le i\le N} |\lambda_i(\tilde{\mathbf{L}}(t))| \qquad \text{for all } t, \tag{5.11}$$

then the consensus reaching of the swarm is guaranteed. Note that the superscript "0" in B_{n}^0 serves as reminder that this analytic derivation was obtained in the absence of intrinsic or extrinsic sources of noise.

This sufficient condition (5.11) on the bandwidth B_{n} reveals the profound connection between, on the one hand the switching communication topology—through the maximum eigenvalue of the normalized directed graph Laplacian $\tilde{\mathbf{L}}(t)$ of the

signaling network, and on the other hand, the necessary information flow for effective swarming measured by the bandwidth B_n. In Ref. [40], it is speculated that the effects of not satisfying this condition (i.e., having $B_n > B_n^0$) can readily be tested experimentally using a simple setup consisting of fish schooling in a tank with a stroboscopic light shining onto them. By reducing the frequency f of the flash, we artificially force the decrease in B_n (and B_e) and we expect that at a given critical frequency $f^c = 2B_n^c$, the coordinated schooling behavior will disappear.

5.3.7 Swarming Collapse Under Data Rate Limitations

Another approach can be used to study the limiting effects of information bottlenecks on swarm dynamics, which consists in carrying out actual SPP simulations. That approach has the advantage of incorporating the combined effects of SNR_n and B_n. More importantly, this simulation approach allows us to carefully study changes in the swarm dynamics in the vicinity of the collapse of long-range order, i.e., the phase transition. That would not be possible with the analytical approach presented in the previous section that focused on preventing a swarming collapse in the absence of noise. It is worth adding that most SPP simulations heretofore reported in the literature have been carried out using arbitrary, yet sufficiently high, bandwidth levels. Therefore, in those past works, the collapse of swarming is rooted in the noisiness of the signaling channel.

We therefore seek evidences of a required minimum information flow by simulating the dynamics of N SPPs governed by (5.4), with decreasing bandwidth B_n—the control parameter—in the presence of different levels of intrinsic noise η_n. Similarly to what has been done in the past chapters, the effectiveness in swarming is measured by the order parameter $\varphi(t) \equiv \frac{1}{N}\left|\sum_{j=1}^{N} e^{i\theta_j(t)}\right|$. For large bandwidths, $B_n \gg B_n^0$, swarms of vastly different sizes systematically produce large-scale order, even in the presence of relatively high noise levels (see Fig. 5.3). For all swarm sizes, the

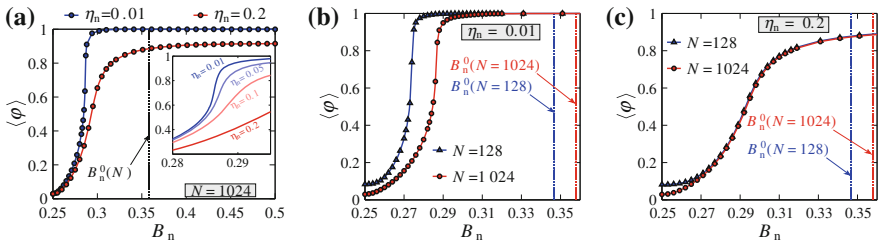

Fig. 5.3 Collapse of swarming with decreasing B_n: **a** $N = 1{,}024$; **b** $\eta_n = 1\,\%$; **c** $\eta_n = 20\,\%$. Values for $B_n^0(N)$ are obtained from (5.11) in the $\eta_n = 0$ limit upon averaging over a sample of 10^4 SSNs ($v_0 = 0.3$, $k = 7$, $\rho = N/L^2 = 100$, and equivalent statistics for all data points. See Sect. 3.2)

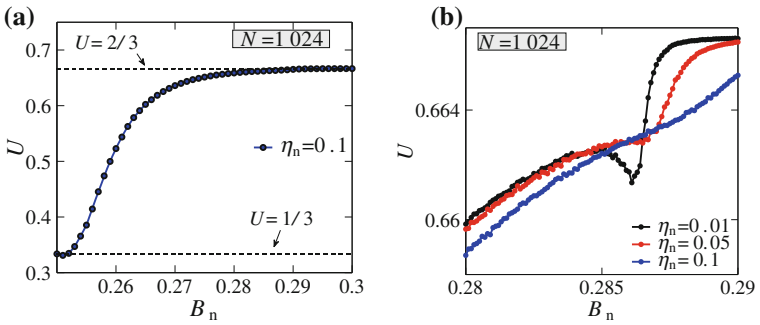

Fig. 5.4 Stationary values of $U \equiv 1 - \langle\varphi^4\rangle/3\langle\varphi^2\rangle^2$: **a** wide B_n range; **b** At the critical bandwidth $B_\mathrm{n}^c = 0.286 \pm 0.001$, U has the same value for not too high η_n ($v_0 = 0.3$, $k = 7$, $\rho = N/L^2 = 100$, and equivalent statistics for all data points. See Sect. 3.2)

simulation results confirm the occurrence of a swarming collapse as expected from our analytical study of the networked sampled-data system (5.9). Indeed, as depicted in Fig. 5.3, a continued reduction in B_n below B_n^0 consistently yields a swarming collapse—corresponding to a disordered state of the system lacking large-scale self-organization—irrespective of the swarm size N or noise level η_n.

These phase transitions are of second order—i.e., they are continuous—since the Binder cumulant (introduced in Sect. 3.3.3) $U \equiv 1 - \langle\varphi^4\rangle/3\langle\varphi^2\rangle^2$, remains positive for all values of the control parameter B_n (see Fig. 5.4a). However, it is very likely that similarly to noise-induced phase transitions, the observation of continuous phase transitions is only apparent owing to strong small-size effects [34]. Indeed, in our particular framework, we are dealing with real-life finite-size swarms. For such swarms, the population N is relatively small, especially compared to the thermodynamic limit, which, as we have seen in Chap. 3, is classically invoked to fully characterize the very nature of a phase transition from the statistical physics standpoint.

We further observe the existence of a transition line for which the critical bandwidth varies with the noise, i.e., $B_\mathrm{n}^c = B_\mathrm{n}^c(\eta_\mathrm{n})$. As expected, the variations of the susceptibility $\chi \equiv L^2(\langle\varphi^2\rangle - \langle\varphi\rangle^2)$ with B_n reveal the occurrence of large fluctuations of the order parameter near the phase transition (see Fig. 5.5). The key point revealed by these simulations concerns the existence of this transition line for which the critical bandwidth varies with the noise, i.e., $B_\mathrm{n}^c = B_\mathrm{n}^c(\eta_\mathrm{n})$. The existence of this transition line could have been anticipated from the interplay between noise and bandwidth originating from the Shannon–Hartley theorem and the expression (5.1) for the channel capacity. Indeed, along the transition line $B_\mathrm{n}^c(\eta_\mathrm{n})$, we have that B_n^c decreases with decreasing η_n. This important observation is consistent with our intuition that a higher noise level would require a higher volume of information to be exchanged for the swarm to self-organize. From the information-theoretic viewpoint, this trend can readily be explained if we assume the existence of a minimum "critical" rate of information R^c below which a collapse of swarming occurs. At the critical point, the max-flow min-cut theorem [29] gives us $C_\mathrm{n} = R^c$ and given expression

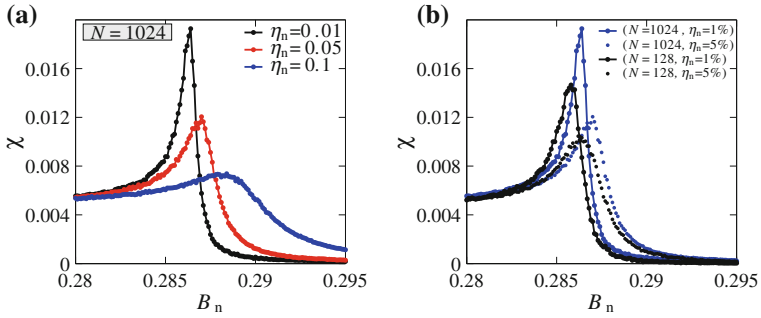

Fig. 5.5 Stationary values of $\chi \equiv L^2(\langle\varphi^2\rangle - \langle\varphi\rangle^2)$: **a** $N = 1{,}024$; **b** $N = 128$ and $N = 1{,}024$. ($v_0 = 0.3$, $k = 7$, $\rho = N/L^2 = 100$, and equivalent statistics for all data points. See Sect. 3.2)

(5.1), we have that $B_{\mathrm{n}}^{c} \downarrow$ with $\eta_{\mathrm{n}} \downarrow$. Despite the singularity in Shannon's capacity at the zero-noise limit, this approach allows us to determine the critical bandwidth in this limit through the intersection of the $U(B_{\mathrm{n}})$ for several nonzero values of η_{n} (see Fig. 5.4b).

5.4 Information and Swarm Design

5.4.1 Acquisition of Stimuli Information by the Swarm

Swarm dynamics is tightly coupled to local signal detection and global information flow throughout the collective. The interaction with the surrounding environment occurs at the agent level where the information extracted from the detected signal is locally processed as what is called stimuli information. The faithful acquisition of stimuli information critically depends on the sensory capabilities of agents—e.g., vision for flocking birds, vision and lateral line sensing for schooling fish, chemo- and mechanosensing for aggregating amoebae. This fact stresses the critical importance of the onboard sensory suite in the design of artificial swarms. Collectives made of units equipped with multiple sensors should therefore be versatile and achieve multifunctionality. However, the integration of stimuli information originating from various channels, and thereby of different nature, remains a significant challenge. For most biological swarms, a full understanding of the integration and interplay of various stimuli remains elusive. An interesting example is given by aggregating amoebae. Their chemotactic behavior is now fairly well understood while we are only starting to grasp their mechanotactic response. The fact of the matter is that we know very little about the amoeba's response to combined mechano- and chemotactic signals. This shows how far biological inspiration can take us with regards to this issue of multimodality integration.

5.4.2 *Dynamic Balancing of Positive and Negative Feedback Loops*

While we have stressed the importance of sufficient and accurate enough stimuli information as the primary source of behavioral information, we have also emphasized the necessity of tracing the swarm information flow. Having identified the underlying transmission channel and studied its properties—as was done in Chap. 4—is necessary but unfortunately not sufficient to fully determine the flow of behavioral information. This issue is particularly acute for biological swarms in which the interplay between signaling network and swarm dynamics is profound. In particular, the topology of the signaling network allows for the establishment of multiple redundant information pathways between any pair of swarming agents. Such feedback loops can either be positive or negative depending on the actual nature of the signal and its origin.

This fact can be better fathomed by analyzing this process within an actual swarm as schematically depicted in Fig. 5.6. Let us focus our attention on agent (1) colored in green, which is receiving behavioral information from neighboring agents. Let us further assume that agents (2) and (3), colored in blue, are detecting an external signal—e.g., oncoming obstacle or threat—and are responding to it thereby triggering meaningful behavioral information in the form of intense fluctuations of the agent's heading represented by arrows in Fig. 5.6. This meaningful information reaches agent (1) directly through interaction links (b) and (e). This direct signal is reinforced by means of positive feedback loops such as (d)–(f), (a)–(k)–(j), (d)–(g)–(h) and many others. At the same time, agent (1) receives behavioral information from the bulk of the swarm represented by red-colored agents such as (4), (5), (6), and (7) for instance. If this swarming behavior yields global order, these agents have very close directions of travel and hence fluctuations of this quantity are quite moderate. This weak signal reaches agent (1) directly through links (f), (h) and (j), but also indirectly through

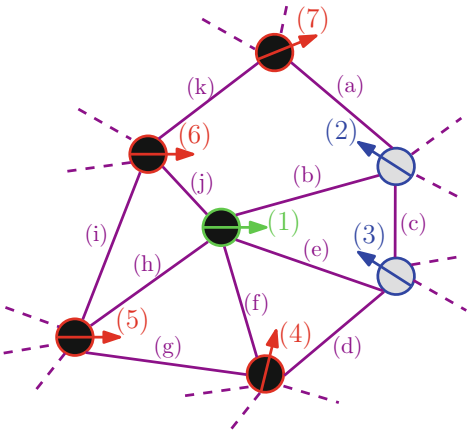

Fig. 5.6 Schematic of a subset of collectively moving agents within a swarm. *Arrows* show the direction of travel and *straight lines* represent the existence of an interaction link between any two agents (considered bidirectional for simplicity)

feedback loops such as (a)–(b), (k)–(j) and many more. These feedback loops do not contribute to a strengthening of the meaningful signal elicited by agents (2) and (3). On the contrary, they weaken it and can therefore be considered as negative feedback loops counterbalancing the amplification of the meaningful signal.

This discussion clearly exposes the meandering of behavioral information with combinations of positive and negative feedback loops. As was already discussed in Sect. 5.2, effective swarming requires a fine balance between these two opposite kinds of feedback loops. Adding to this the important fact that those loops are dynamic helps us appreciate the level of complexity in the flow of information within natural swarms. This dynamic character stems from the dynamic nature of the transmission channel, which has both pros and cons. On the one hand, it provides opportunities for adaptivity, which in general is beneficial to swarm dynamics. On the other hand, if not properly managed, it can lead to a reduction, or even a loss, of connectivity or a reduced capacity of information, resulting in adverse effects and a possible collapse of swarming. These effects are common to most spatially embedded networks such as transport networks—the power grid, the internet, road networks, etc.—which are prone to the emergence of jamming and congestion. These undesirable phenomena have already be encountered in Sect. 3.3.4 when studying the nonequilibrium system character of swarms. Solé has discussed the connection between information flow, traffic jams and phase transitions in Ref. [41], with particular emphasis on internet traffic and human agents. Some unexpected similarities and analogies exist.

Owing to their evolutionary-optimized character, biological swarming agents such as fish and birds do not naturally exhibit the bandwidth-induced swarming collapse reported in the previous sections. Beyond the consequences for self-organized biological systems, having sufficient information signaling capacity is key to the design of artificial swarms. Furthermore, a complete understanding of this dynamic balancing of feedback loops is lacking. The swarm designer is therefore left with several possible conservative options when designing the transmission channel such as for instance: (i) maintaining a static interaction network, or (ii) imposing a uniform interaction rule as is the case with the k-nearest neighbor rule modeling topological interactions in flocks of starlings. These options are most likely suboptimal but may help in preventing a swarming collapse due to breakdown in behavioral information flow.

5.4.3 Leveraging Technological Advances for Novel Swarm Designs

Explosive advances in the field of sensory and communication devices—primarily with Miniaturization, extended range and increased accuracy—coupled to a still ongoing revolution in information processing technologies are generating tremendous opportunities to develop unparalleled swarming technologies. However, it is important keeping in mind scalability issues when designing swarms. Indeed,

designing a swarming agent with a costly suite of sensors and communication devices would somehow amount to a technological nonsense. Swarms function and solve problems by drawing on masses of relatively simple elements. This power of masses follows nontrivial scalability laws specific to each swarming system, but in general demands at least several units. Thus, the sharp drop in unit prices for most of the common robotic sensors and transceiver modules constitute yet another favorable element for the development of artificial swarming systems. Although this last consideration is of economic nature, it can prove to be as important as other technological factors.

5.4.4 Coupling Between Information Flow and Agent's Movement

In the previous sections, we have repeated how the dynamic nature of the transmission channel is the source of intricate flow of behavioral information in swarms. This underscores how information flow and signaling network dynamics are deeply intertwined. In Chap. 4, the network dynamics was shown to be rooted in the physical movement of the swarming agents—as was discussed, the signaling network is embedded in space with nodes moving according to a specific update rule. The coupling between information flow and agent's movement is therefore apparent.

In Ref. [42], Sumpter argues that: "the key to understanding collective behavior lies in identifying the principles of the behavioral algorithms followed by individual animals and of how information flows between the animals." However, the analysis presented in this chapter uncovered the actual interplay between behavioral algorithm and information flow. This important aspect therefore prevents a design process of both of these fundamental elements separately and instead requires a holistic approach combining: (i) physical movement, (ii) dynamics of the signaling network, and (iii) information flow through multiple positive and negative feedback loops. To fully close the loop, one needs to connect (iii) and (i) and that is achieved through local decision-making processes based on information processing. This final element will occupy us throughout the next chapter.

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Chapter 6
A Computational Approach to Collective Behaviors

In her book "Information Theory and the Living System," Gatlin [1] states that "Life may be defined operationally as an information processing system—a structural hierarchy of functioning units—that has acquired through evolution the ability to store and process the information necessary for its own accurate reproduction." Somehow this statement can readily be adapted to define swarms from the information processing viewpoint: a swarm can be defined operationally as a single distributed information processing system—a dynamic and decentralized structure of functioning units—that has the ability to process information and adapt to changing environments. This definition echoes the general role of computation in complex systems given by Mitchell [2]: "computation is what a complex system does with information in order to succeed or adapt in its environment."

This chapter discusses how the flow of social information is processed by the entire swarm in what essentially amounts to collective computation, thereby generating the individual response of each agent in the physical space. If self-organization is at play, a bird's eye view of the collective response in the physical space should reveal large-scale patterns of long-range and time-persistent correlations as was presented in Chap. 3.

6.1 From Collective Behavior to Computation and Information Processing

As already mentioned in past chapters, the sight of large—and not so large—numbers of animals moving together in unison has been an inexhaustible source of inspiration and enquiry for generations. In the last two decades, a large body of research has focused on gaining insight into some of the key elements underlying collective animal behavior: e.g., decision-making, adaptation to dynamic environmental conditions, structures, and regulation [3]. The connection with complex systems science is now apparent and with that comes a better understanding of self-organizing emergent behaviors, such as birds flocking, fish schooling, locusts marching, amoebae

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aggregating, and humans crowding [4–7]. Furthermore, some recent field and laboratory experiments have provided scientists with large-scale and high-resolution data sets [8–10]. The postprocessing and analysis of these highly resolved kinematic samples allow for a better understanding of the local interactions among individual agents within bird flocks or fish schools—what Sumpter calls the behavioral algorithm [11], and provide us with the required knowledge to explicitly construct the swarm signaling network (see Chap. 4). With such explicit construction of the dynamic communication channel, we have been able to study the importance of the social transfer of information on the effectiveness of collective behaviors (see Chap. 5).

6.1.1 Nature of Information and Its Storage in Swarms

Adaptation, collective decision-making and learning, all amount to some form of information processing, or computation [12]. Indeed, in swarms, it requires all individuals to go through a behavioral change of state. Let us exemplify this abstract concept of information processing for a school of fish, or for that purpose any collectively traveling group of agents, which we model using our minimalist SPP description introduced in Sect. 3.2 and repeatedly used in past chapters. For this system, the information is distributed across each and every agent of the swarm and corresponds to the configuration of the states of agents—the state variable ψ_i of a given agent i, the fluctuation $\psi_i' = \psi_i - \langle \psi_i \rangle_{\text{swarm}}$ of the state variable, or possibly other quantities derived from ψ_i. An alternative way of looking at such distributed information would be to adopt a statistical approach, namely saying that information takes the form of dynamic statistics of quantities derived from the set of state variables. Such a statistical approach emerged naturally in Chap. 3 within the statistical physics framework (see Sect. 3.3). The input condition corresponds to the initial states of agents (see top left box in Fig. 6.1), while the output is the result of collective information processing shown in the bottom left box in Fig. 6.1, namely when an updated set of values for the states of agents is being generated, thereby closing the loop in Fig. 6.1. At each time step—given our discrete-time approach, an interaction network, or communication channel is established (see top right box in Fig. 6.1), with a topology that depends on the states of agents. As was seen in Chap. 5, this communication channel enables a flow of social information throughout the swarm as is schematically shown in the bottom right box in Fig. 6.1. As highlighted by Mitchell [2], one consequence of encoding the social and distributed information as dynamic statistics of local variables is that no individual agent has access to the global state of the system. Local information exchanges depicted in the bottom right box in Fig. 6.1 are therefore based on spatial and temporal sampling, which is in complete agreement with some recent analyses and studies of social information transfer presented in Sect. 5.3.2.

Although this looped sequence of events is really helpful to improve our understanding of the various processes at play, it unfortunately oversimplifies and misrepresents a more complex reality in which all these processes run concurrently and commingle with one another.

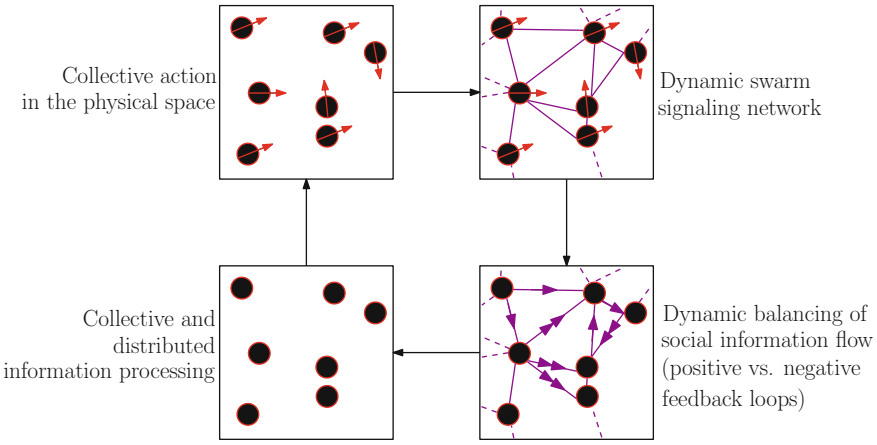

Fig. 6.1 Schematic of the four key elements governing the dynamics of swarming systems: (*i*) Collective action in the physical space (see Chap. 3), (*ii*) Dynamic swarm signaling network representing the interaction network, which serves a communication channel (see Chap. 4), (*iii*) Social information flow with dynamic balancing of positive and negative feedback loops (see Chap. 5), (*iv*) Collective and distributed information processing or computation (this chapter). In this figure, all four elements are shown acting on one another sequentially but in reality the associated processes are completely intertwined and run concurrently

6.1.2 Swarms and Algorithms

This algorithmic decomposition of some collective behaviors is nothing new. For almost two decades now, computer scientists have been quite successful at mimicking some of these collective behaviors, in particular those occurring in insect colonies. They did that not based on any entomological interest but instead to devise very efficient optimization algorithms. That led to the development of now well-known algorithms, such as swarm particle optimization methods, ant colony optimization techniques, etc. [13]. Some of these algorithms have had game-changing consequences in some industrial sectors such as network routing and logistics.

Strikingly, entomologists have reversed roles and are using tools from computer science to deconstruct behavioral patterns of social insects in order to explicitly identify behavioral algorithms routinely used by those insects [14]. Beyond the insight gained into the collective behaviors of insects, this approach is revealing of the importance that collective information processing plays in the dynamics of swarms. For instance, most social organisms involved in collective behaviors discussed in the past chapters are obviously capable of collective decision-making but also of collective learning [15]. From insects to quadrupeds, animal groups need to collectively fine tune vital decisions, such as nesting and foraging sites, communal migration routes, predator avoidance strategies, etc. [16]. The effectiveness of the collective decision-making process can be improved by imitating the behavior of better-informed or more knowledgeable neighbors across many taxa, including insect colonies [17].

However, there are still very many key questions related to the effectiveness of collective decision-making that remain to be elucidated [18, 19], and for which a computational approach could help by introducing new concepts and a new toolbox that may offer completely new vantage points. However, this optimistic perspective should be counterbalanced by the actual fact that up to now, a high-level description of information processing in natural systems is still lacking [2].

6.2 The Theory of Computation

For us to better understand and appreciate the importance and relevance of computation for the dynamics and control of swarming systems, it is necessary to clearly define what we really mean—in scientific terms—by computation, and also what can be computed. In our times where computation pervades our everyday life and where computers are hidden or embedded in an ever-increasing number of devices around us, from smartphones to cars, one could think that computation is simply about excessively fast additions and multiplications of zeros and ones. Even if this simplified vision of computation is partially correct, it fails to capture the wider meaning of computation. A classical quote often heard in Computer Science is that "Computation is no more about computers than astronomy is about telescopes."

The objective of this section is not to give a complete introduction to the theory of computation (interested readers are referred to the classic monograph by Sipser [20] or other similar general references) but instead to briefly review some of the key concepts about information processing and computation that are directly relevant to our discussion about swarming systems.

6.2.1 A Definition of Computation

In the most general terms possible, computation occurs whenever information is processed: searching for a possible meaning in the acquired information, storing it, and transforming this information according to a given algorithm. The outcome of information processing takes the form of results or actions depending on whether the system is embedded in the virtual or the physical world respectively. In the particular case of swarming systems, the acquired information is a combination of sensory information and behavioral information, as was discussed in Chap. 5, and the outcome is an action corresponding to the individual response of a given agent, such as the change in its direction of motion for collectively traveling agents.

6.2.2 The Concept of Computability

One of the central questions originating from the above general definition of computation is associated with the key concept of *computability*, or in other words, whether

for a given problem, there exists a set of commands, i.e., an algorithm, that can solve it. An even more general question at the core of the theory of computation is whether anything can be computed, and if not, what can we know about the associated limits to computing. As Sipser puts it: "What are the fundamental capabilities and limitations of computers?" [20].

Having defined computability, one may wonder about the relevance of this concept with respect to collective behaviors. This question can be seen as totally irrelevant for biological swarms since those systems are obviously performing collective information processing leading to an emergent behavior bringing along the way some benefits to the group. On the contrary, the concept of computability is central to the design of artificial swarms. Indeed, if one is tasked with devising a swarming system meant to perform a specific task—e.g., the control of microscopic robots in biomedical applications, collective surveillance by mobile sensors, etc.—then the designer is faced with the formidably difficult question of the existence of an actual swarming algorithm capable of generating a self-organizing behavior that will more or less efficiently accomplish the required task. As was already mentioned in Sect. 2.3, such design problems fall into the class of inverse problems, for which existence proofs often remain elusive. Therefore, the concept of computability could prove to be of theoretical interest for the swarm designer, but unfortunately of limited practical interest. However, one should keep in mind that even if a design is obtained—i.e., a swarming algorithm has been devised, it is not guaranteed that the latter will not exhibit catastrophic behaviors when confronted with pathological situations.

6.2.3 Computation and Causal Systems

Computation is also closely related to the physical concept of causality, which is central to the study of dynamical systems (see Chap. 3 and in particular Sect. 3.4). Indeed, there exists an equivalence between computation and deterministic time evolution. Specifically, one of the aims of computation theory is to find a representation for all possible discrete deterministic systems—also known as causal systems. From that point of view, all computations can be seen as causal relationships. This important fact constitutes one of the few links bridging the gap between information theory and physics. Another such link highly relevant to swarm dynamics is the relation between phase transitions in Boolean networks, criticality and phase transition in collective computation [21].

Further to this equivalence between computation and causality, one should be aware of the fact that certain classes of deterministic dynamical systems are capable of the property known as universal computation [22]. This property was already introduced in Sect. 3.1.1, when discussing elementary cellular automata, and specifically with rule 110, which has been proved to be Turing complete, that is a universal computer. This central point about certain classes of deterministic dynamical systems being Turing complete implies that possibly many different computational models or problem formulations happen to belong to the same equivalence class of computation, which means that they can theoretically simulate one another.

6.2.4 *Randomized Algorithms, Probabilistic Turing Machines, and Computation*

Although the discussion in the previous section is an essential step in building a theory of computation, its direct relevance to collective behaviors is clouded by the probabilistic nature of swarm dynamics. As was discussed in all past chapters, the existence of noise and the ensuing correlated fluctuations are central to the effective functioning of swarms. This includes keeping the dynamics of the swarm at the edge of chaos, i.e., close to criticality. Note that it has been proposed that most biological systems may be poised near criticality [23], that is, they reside near the critical point between an ordered and a disordered phase, where the system becomes highly correlated. This has been proven to be the case for swarms of midges, which are lacking global collective order [24]. Living on the edge of disorder seems to be a favorable strategy for collectives faced with changing environments.

The probabilistic nature of swarm dynamics is therefore better approached using the concept of randomized algorithms from the computability theory. A randomized algorithm is an algorithm that employs a degree of randomness as part of its programmatic logic. In the case of swarms, this randomness is a direct consequence of the probabilistic aspect of behaviors at the local level. Classical randomized algorithms typically use uniformly random variables as an auxiliary input to guide their behaviors, in search for good performance in some global statistical sense. This latter point resonates with the alternative way of looking at social information in swarms based on dynamic statistics of quantities derived from the set of state variables (see Sect. 6.1).

From a theoretical standpoint, randomized computation is built upon the concept of probabilistic Turing machine, which is a nondeterministic Turing machine that randomly chooses between the available transitions at each point according to some probability distribution. Without getting into any further detail, it is worth knowing that these important notions are the fruits of decades of research crowned by the groundbreaking works of Kurt Gödel, Alan Turing and Alonzo Church among others [2].

6.3 Collective Information Processing in Swarms

In the past five chapters, we have repeated, albeit using different terms and in different frameworks, the central feature of swarms, namely that through an emergent process of self-organization, a collection of interacting agents can perform actions that are far beyond each single agent's ability. In the past sections, we have discussed how such "augmented" collective actions is the outcome of distributed and collective information processing of the social information acquired by individual agents. As was seen in Chap. 5, the information accessible to each agent varies greatly depending on the agent's position within the swarm and on other environmental factors

external to the swarm. Typically, this local information may be limited, inaccurate, or even absent if the swarm signaling network is not connected. If the informational bottlenecks presented in Sect. 5.3.5 are avoided, everything appears to be functioning as if individual agents have access to global knowledge of the environment, the so-called augmented perceptual range.

6.3.1 A Tentative Classification of Collective Decision-Making Processes

It has been reported that most classical collective behaviors encountered in social and natural systems can fall under three broad categories of information processing or computation [12]: (i) sorting, (ii) optimization, and (iii) consensus reaching. Although these three categories match a wide range of empirical evidences, they can somehow be found to belong to the single even broader class of decision-making problems. In the case of sorting (resp. optimization and consensus reaching), the decision is the about a sorting (resp. optimization and consensus reaching) process. The literature in the field of decision-making is extremely rich [18, 19] but the field lacks a common lexicon owing to its study in vastly different disciplines: social sciences, biology, engineering sciences, etc. Therefore, instead of trying a synthesis under the common framework of decision-making, we will summarize the key aspects of the above three categories following the analysis reported in Ref. [12].

Sorting is a dynamic process, which through amplification of certain pathways achieves a classification of various emergent options available. This amplification of certain pathways is central to this process and is made possible thanks to the existence of dynamic positive feedback loops (see Sects. 5.2 and 5.4.2).

Optimization just like sorting is a dynamic process. The very sophisticated social organizations in insect colonies—e.g., ants foraging, division of labor in termites mounds, stop signaling providing cross inhibition in honeybees, etc.—perfectly epitomizes that concept of collective optimization. It is therefore no wonder that these collective processes have been such a unique source of inspiration for mathematicians, computer scientists, and operations researchers alike. In general, the convergence towards an optimal (often suboptimal) collective choice or action rests upon a dynamic search in the space of all possible choices or actions. Similarly to sorting, this dynamic search is not guided by any central force but instead emerges out of the amplification of certain pathways in the search space.

Consensus reaching is the collective action that was the most extensively discussed and studied throughout this book since we emphasized swarms of collectively moving units. As discussed in past chapters, such swarms—like school of fish, group of cockroaches, band of locusts, and flock of birds for instance—are characterized by high and robust levels of global alignment, which confer them a great ability to stay together and act in unison, thus leading to several functional benefits at the group level. In the field of engineering control, achieving consensus has a clear meaning:

it is the convergence to a common state asymptotically or in a finite time among all group members through local interactions, as stated in Sect. 4.4. However, the interesting case of swarm of midges studied by Attanasi et al. [24] revealed how collectively moving insects are swarming in the absence of global order. Consensus reaching has also been extensively studied by social, and even political scientists given its importance for human social organization. In his book titled "The wisdom of crowds," Surowiecki [25] discusses at length the incredible effectiveness of groups in achieving consensus points using an efficient collective integration of information. It is interesting to note, that even noninteracting collective—i.e., non-swarming systems—are given a statistical edge in delivering a group decision that surpasses the average individual decision, as we long know since the work by Condorcet [25]. Interestingly, some recent works have shown that in some complex environments, collective decision accuracy—of the consensus reaching type—can be maximized by small group sizes [26] or by the presence of conflicting signals [19]. It is worth adding that very little is known about the influence of the group organization or its interaction network on the effectiveness of consensus reaching [27].

It is fair to say that collective information processing is probably the element of swarming systems that is the least well understood even though its importance is acknowledged. We speculate that this might be due to the vast differences in individual behavior, sensory integration and response of living units that compose different swarms. In other words, collective computation by swarms might be overly sensitive to the characteristics of the decision-making apparatus at the agent level. Indeed, the most effective and varied collective information processing tasks observed empirically are all coming from swarms of living units. In comparison, collective behaviors in physical and chemical systems—made of interacting but nonliving units—perform rather primitive collective computation.

6.3.2 The Importance of Randomness for Collective Computation

We just saw in the previous section that the biological complexity of information processing at the individual level seems to be at the root of the complexity of information processing at the swarm level. Irrespective of the veracity of this statement, there is a central consequence associated with the complexity of decision-making at the agent level: biological variability induces a certain level of stochasticity. This randomness has been shown to play a key role in the effectiveness of swarm systems. Here, our understanding of probabilistic computational algorithms—e.g., Monte Carlo algorithms, simulated annealing—allows us to appreciate the fact that stochasticity enables an exploration of the immense space of actions for swarming agents. Note that even for very large swarms, the number of agents acting collectively is still formidably small as compared to the number of options in the space being explored. Furthermore, as was discussed in Sect. 6.1, for swarms, the information to be

collectively processed takes the form of dynamic statistics of quantities derived from the set of state variables. Consequently, a stochastic approach for collective computing appears all the more suited. Finally, as rightly pointed out by Mitchell in Ref. [2]: "randomness must be balanced with determinism: self-regulation in complex adaptive systems continually adjusts probabilities of where the components should move, what actions they should take, and, as a result, how deeply to explore particular pathways in these large spaces." This last point relates to the existence of some form of determinism in most probabilistic algorithms. For instance, in classical Monte Carlo algorithms, the computation of inputs is fundamentally deterministic while these inputs are intrinsically of stochastic nature.

6.4 Computation and Swarm Design

In this section, we would like to ponder over some key points about collective information processing in the framework of the design and control of swarming systems:

- *Simplicity versus complexity at the individual level*: in previous chapters, we strongly advocated resorting to simple agents so as to facilitate the design and control of swarm dynamics. Indeed, we saw on many occasions that various effective swarming dynamics can be achieved with extremely simple agents. However, in this chapter, we came to the realization that some specific collective behaviors require a certain level of complexity at the agent's level. Note that this realization does not prove wrong or contradict any of the proposed design principles enumerated in past chapters. However, this realization stresses the need to develop a specific set of design principles for a lower hierarchical level of complexity in swarms, namely the sensory cascade—detection of sensory signals, information processing at the agent level, followed by response (see Sect. 5.3.3). Given how arduous is the design of swarming systems from the bottom up—i.e., the upper level of complexity corresponding to the behavioral algorithm at the agent's level, there is no doubt that a complete design of both levels, including their coupling, represents a formidably challenging task.
- *Collective cognition*: this aspect of collective behaviors has been deliberately left out of this book. This choice was certainly not because of the importance of collective cognition to the field, but mainly because it is a nascent field of research lacking maturity although some interesting studies are starting to appear [15].
- *Randomness and statistics*. It has been argued that the statistic nature, both in space and time, of the sensory information and behavioral information, coupled to a certain "dose" of stochasticity is responsible for the effectiveness and accuracy of the swarm's response to changing environments. This therefore raises the question of the pertinence of mimicking such ways of functioning for small swarms—i.e., for swarms with sizes small enough that a statistical approach becomes irrelevant. For such small swarms, a purely deterministic approach for causal systems seems more promising and appropriate.

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Chapter 7
Outlook: Can Swarms Be Designed?

Swarm intelligence offers a unique alternative way of designing "intelligent" systems, in which the combination of autonomy, emergence (through self-organization), and distributed problem solving replaces embedded centralized control. In these concluding words, it is no longer necessary to discuss the power and distinctive capabilities of swarms in collectively solving complex tasks and problems. The ultimate goal is therefore to leverage the power, robustness, flexibility, and scalability of swarms while still being able to design, direct, and control the collective task to be performed. In the face of changing circumstances, possibly adverse external factors or internal element failures, a robust artificial swarm should continue to operate if properly designed. Furthermore, a flexible artificial swarm should, in principle, be able to maintain its distributed operation through graceful degradation. In other words, if properly designed, an artificial swarm should avoid catastrophic collapses in collective action. The property of scalability should not be understated given the recent explosion in low-cost, miniaturized and highly reliable sensors, communication devices, and microcomputers [1].

Finally, the question "Can swarms be designed?" might appear as completely irrelevant after several chapters discussing possible design principles for swarming systems. Even though self-organization pervades the natural world, engineers are only starting to consider it as a central element in the design process aimed at devising swarming systems. Moreover, as rightly pointed out by Prokopenko [2], the process of self-organization, which is at the core of swarming, is not an element that can be naturally integrated into a traditional and methodical design process.

If one turns again to Mother Nature in search of general design principles for self-organization, it will appear that evolution is the key driving element. The social agents—fish, birds, ants, locusts, cockroaches, amoebae, etc.—have all developed their unique swarming capabilities through an evolutionary process, which is not fully understood and which can hardly be faithfully replicated as part of a design process. There is no doubt that the future of swarm design should in itself be an emergent process: beyond self-organization, we could envision a *self-design process*.

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At this point, *self-design* remains a vision of the future, yet more pragmatic and realistic design processes can still be achieved. For instance, the advances in the field of evolutionary computing serve as an encouragement toward the possibility to mimic evolutionary processes for the design of the behavioral algorithm, the swarm signaling network, and the dynamic balancing of feedback loops. Since the power of coupling emergence and evolution does not appear to be naturally integrable into design processes, the best approach at this point seems to be a modular approach consisting in advancing the design of the vital components in a swarming system: sensing system for the dynamic environment, improved communication channel between swarming agents, effective information flows throughout the swarm, and advanced collective information processing or computation. That was the approach underpinning the successive chapters of this book and this book as whole. We hope that after reading these last words, you will be compelled to start designing your own swarm!

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